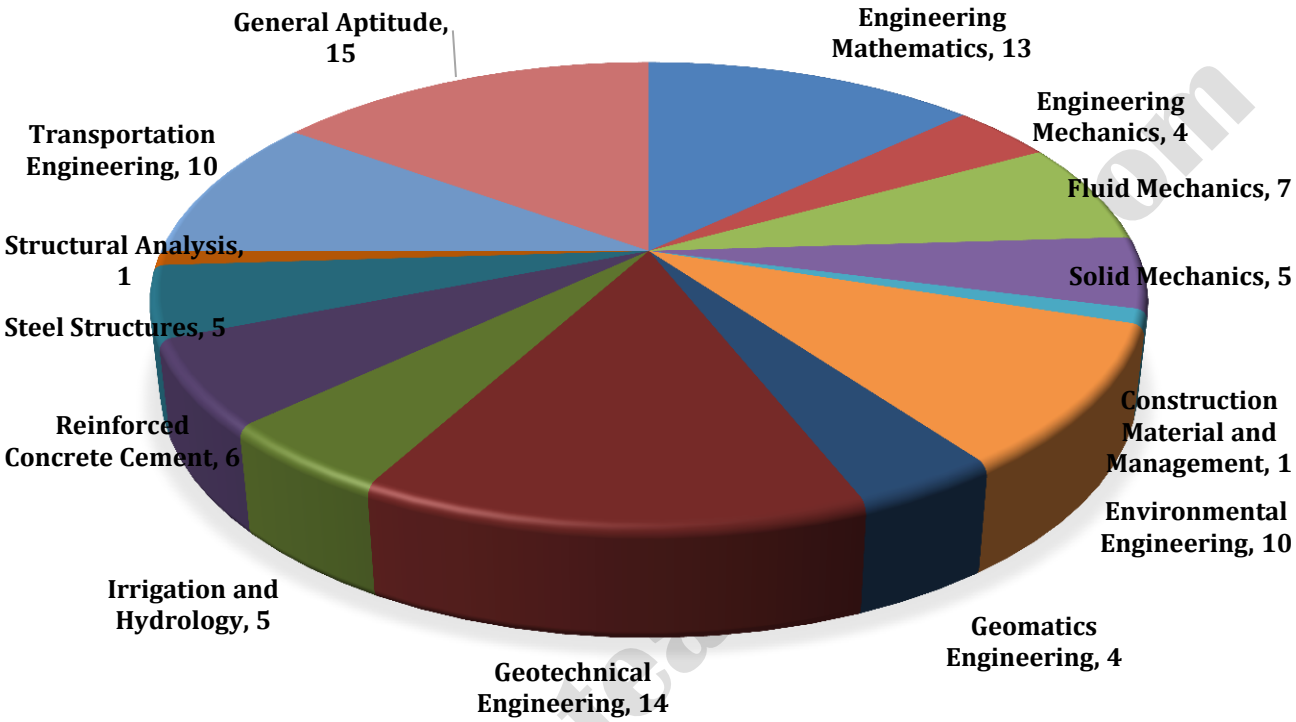
ANALYSIS OF GATE 2018

Civil Engineering



CE ANALYSIS-2018_11-Feb_Afternoon

SUBJECT	No. of Ques.	Topics Asked in Paper(Memory Based)	Level of Ques.	Total Marks
Engineering Mathematics	1 Marks: 5 2 Marks: 4	Eigen Vector; Statistics	Tough	13
Engineering Mechanics	1 Marks: 0 2 Marks: 2	Trusses and Frames	Medium	4
Fluid Mechanics	1 Marks: 1 2 Marks: 3	Boundary Layer; Energy Depth Relationship; Discharge calculation	Tough	7
Solid Mechanics	1 Marks: 1 2 Marks: 2	Simple Stress and Stain; Shear Force and Bending Moment; Stresses in Beams Deflection of Beams;	Medium	5
Construction Material and Management	1 Marks:1 2 Marks: 0	Building Materials	Medium	1
Environmental Engineering	1 Marks: 2 2 Marks: 4	Flocculation; BOD; Chemical Characteristics	Medium	10
Geomatics Engineering	1 Marks: 0 2 Marks: 2	Theodolite and Traversing; Leveling;	Easy	4
Geotechnical Engineering	1 Marks: 4 2 Marks: 5	Compression Index; Active Earth Pressure, Permeability	Medium	14
Irrigation and Hydrology	1 Marks: 3 2 Marks: 1	Direct Run-Off (DRH); Ground Water Technology; Delta and Duty; Occurrence of sludge	Medium/Easy	5
Reinforced Cement Concrete	1 Marks: 2 2 Marks: 2	Single Reinforced Beams	Medium	6
Steel Structures	1 Marks: 1 2 Marks: 2	Welding Connection & Plastic Analysis	Medium	5
Structural Analysis	1 Marks: 1 2 Marks: 0	Slope And Deflection Of Structure	Medium	1
Transportation Engineering	1 Marks: 4 2 Marks: 3	Overtaking; Sight Distance	Medium/Easy	10
General Aptitude	1 Marks: 5 2 Marks: 5	Clocks, Proportion, Vocabulary, Combinations, Logarithms and Equations	Tough	15
Total	**65**			**100**
Faculty Feedback	Majority of the question were concept based. General Aptitude And Mathematics is Very Easy. Core Subject Questions were 50% easy, 30% medium and 20% tough.			

GATE 2018 Examination

Civil Engineering

Test Date: 11-Feb-2018

Test Time: 2:00 PM 5:00 PM

Subject Name: Civil Engineering

General Aptitude

Q.1 - Q.5 Carry One Mark each.

1. A three member committee has to be formed from a group of 9 people. How many such distinct committees can be formed?

(A) 27 (B) 72

(C) 81 (D) 84

[Ans. D*]

$$^{9}C_{3} = \frac{9!}{6! \times 3!} = \frac{9 \times 8 \times 7}{6} = 84$$

2. $\underbrace{(a + a + a + \cdots\cdots\cdots + a)}_{n \text{ times}} = a^2b$ and $\underbrace{(b + b + b + \cdots\cdots\cdots + b)}_{m \text{ times}} = ab^2$, where a, b, n and m are natural numbers.

What is the value of

$$\underbrace{(m + m + \cdots\cdots\cdots + m)}_{n \text{ times}} \times \underbrace{(n + n + \cdots\cdots\cdots + n)}_{m \text{ times}} ?$$

(A) $2\,a^2b^2$ (B) a^4b^4

(C) $ab(a + b)$ (D) $a^2 + b^2$

[Ans. B*]

$$\underbrace{(a + a + a + \cdots\cdots\cdots + a)}_{n \text{ times}} = na = a^2b \ldots\ldots (i)$$

$n=ab$

$$\underbrace{(b + b + b + \cdots\cdots\cdots + b)}_{m \text{ times}} = mb = ab^2 \ldots\ldots (ii)$$

$$\underbrace{(m + m + \cdots\cdots\cdots + m)}_{n \text{ times}} \times \underbrace{(n + n + \cdots\cdots\cdots + n)}_{m \text{ times}}$$

$mn \times mn = (mn)^2$

from (i) and (ii) $mn = a^2b^2$

So result, $(mn)^2 = (a^2b^2) = a^4b^4$

3. For non-negative integers, a, b, c, what would be the value of $a + b + c$ if $\log a + \log b + \log c = 0$?

(A) 3 (B) 1

(C) 0 (D) – 1

[Ans. A*]

As a, b, c are non-negative integers and given $\log a + \log b + \log c = 0$
$\log(a \times b \times c) = \log 1$
$\Rightarrow a \times b \times c = 1$
Which can be possible for simple values?
$a = b = c = 1$
Hence $a + b + c = 1 + 1 + 1 = 3$

4. "Although it does contain some pioneering ideas, one would hardly characterize the work as _____."
The word that best fills blank in the above sentence is
(A) Innovative (B) Simple
(C) Dull (D) Boring
[Ans. A*]
Innovative is similar to poineer.

5. "His face _____ with joy when the solution of the puzzle was _____ to him."
(A) Shone, Shown (B) Shone, Shone
(C) Shown, Shone (D) Shown, Shown
[Ans. A*]
Shone - It is past - participle and past form of shine.
Shown - To show means to reveal and point out something.

Q.6 - Q.10 Carry Two Mark each.

6. A faulty wall clock is known to gain 15 minutes every 24 hours. It is synchronized to the correct time at 9 AM on 11th July. What will be the correct time to the nearest minute when the clock shows 2 PM on 15th July of the same year?
(A) 12:45 PM (B) 12:58 PM
(C) 1:00 PM (D) 2:00 PM
[Ans. B*]
9 Am of 11 July of 2 PM on 15th July =101 hours
$\left(24 + \frac{15}{60}\right)$ hours of incorrect clock $= 24$ hours of correct clock
$\left(24 + \frac{15}{60}\right)$ hours of IC $= 24$ hours of correct clock
1 hour of IC $= \frac{96}{97}$ hours of correct clock
101 hour of IC $= \frac{96}{97} \times 101$ hours of correct clock
$= 99.958$ hours of correct clock
$= 99$ hours $+ 0.95876 \times 60$ minutes of correct clock
$= 99$ hours $+ 57.525$ minutes
$= 99$ hours and approx 58 minutes
So correct time will be
2PM, 11th July +(99 hours and 58 minutes)=12:58 OM on 15th July

7. Given that $\frac{\log P}{y-z} = \frac{\log Q}{z-x} = \frac{\log R}{x-y} = 10$ for $x \neq y \neq z$, what is the value of the product PQR?

(A) 0 (B) 1

(C) xyz (D) 10^{xyz}

[Ans. B*]

$\log P = 10(y - z)$

$\log Q = 10(z - x)$

$\log R = 10(x - y)$

$\log P + \log Q + \log R = 0$

$\log(PQR) = \log 1$

$PQR = 1$

8. In manufacturing industries, loss is usually taken to be proportional to the square of the deviation from a target. If the loss is Rs. 4900 for a deviation is 7 units. What would be the loss in Rupees for a deviation of 4 units from the target?

(A) 400 (B) 1200

(C) 1600 (D) 2800

[Ans. C*]

Loss $= kd^2$ for duration of 7 units

$4900 = k(7)^2 \Rightarrow k = 100$

Loss $= kd^2$ For duration of 4units

$= k(4)^2 \Rightarrow 16\,k = 1600$

9. Each of the letters in the figure below represents a unique integer from 1 to 9. The letters are positioned in the figure such that each of (A + B + C), (C + D + E), (E + F + G) and (G+ H + K) is equal to 13. Which integer does E represent?

A	B	C			
		D			
		E	F	G	
				H	
				K	

(A) 1 (B) 4

(C) 6 (D) 7

[Ans. B*]

$A + B + C = C + D + E = E + F + G = G + H + K = 13$

If we add all, we will get $= 4 \times 13 = 52$

But sum of all natural number 1 to $9 = 45 = 9 \times 10/2$

$A + B + C + C + D + E + E + F + G + G + H + K = 52$ ... (i)

$A + B + C + D + E + F + G + H + K = 45$ ... (ii)

Subtraction eq. (ii) from (i)

Hence, $C + E + G = 7$ ... (iii)

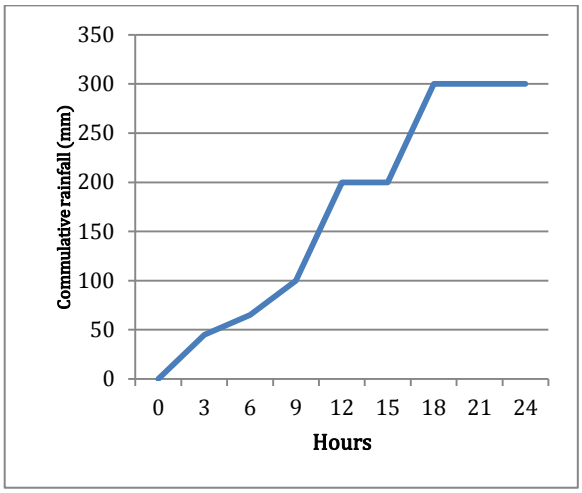
Also, $C + D + E = 13$ ... (iv)

Subtraction eq. (iii) from (iv)

$D - G = 6$

$E = 4$

10. The annual average rainfall in a tropical city is 1000 mm. On a particular rainy day (24 hours period), the cumulative rainfall experienced in the city is shown in the graph. Over the 24-hours period, 50% of the rainfall falling on a rooftop, which had an obstruction free area of 50 m^2, was harvested into a tank. What is the total volume of water collected in the tank in liters?



(A) 25,000
(B) 18,750
(C) 7,500
(D) 3,125

[Ans. C*]

Cumulative rainfall=300 mm

$$50\% \text{ of rainfall} = 300 \times \frac{50}{100} = 150 \text{ mm}$$

Area $= 50\ m^2$

$\Rightarrow$ Volume stored in tank $= 150 \times 10^{-3} \times 50\ m^3 = 7500\ l$

Technical

Q.1 - Q.25 Carry One Mark each.

1. A reinforced-concrete slab with effective depth of 80mm is simply supported at two opposite end on 230 mm thick masonry walls. The center to center distance between the walls is 3.3m. As per IS 456:2000, the effective span of the slab (in m, up to two decimal places) is _____

[Ans.*] Range: 3.15 to 3.15*

Effective depth

$d = 80$ mm

Width of support $= 230$ mm

C/C distance between walls $= 3.30$ m

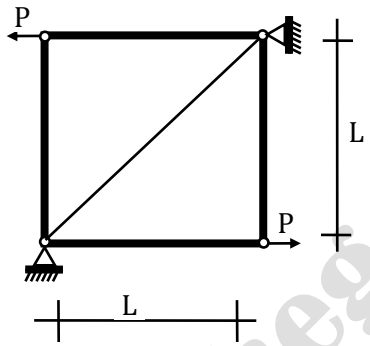
Clear span of slab $= 3.30 - 0.23 = 3.07$ m

$$\text{Effective span} = \text{Minimum}\begin{cases} L_{clear} + d \\ \text{C/C distance between supports} \end{cases}$$

$$= \text{Minimum}\begin{cases} (3.07 + 0.08 = 3.15 \text{ m}) \\ 3.3 \text{ m} \end{cases}$$

$L_{eff} = 3.15$ m

2. All the member of the planar truss (See figure). Have the same properties in terms of area of cross-section (A) and modulus of elasticity (E).



For the loads shown on the truss, the statement that correctly represents the nature of forces in the members of the truss is

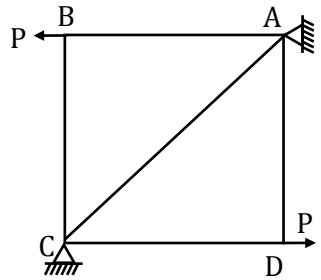
(A) There are 3 members in tension, and 2 members in compression

(B) There are 2 members in tension 2 members in compression , and 1 zero –force member

(C) There are 2 member tension. 1 member in compression and 2 zero-force members

(D) There are 2 member in tension, and 3 zero-force members

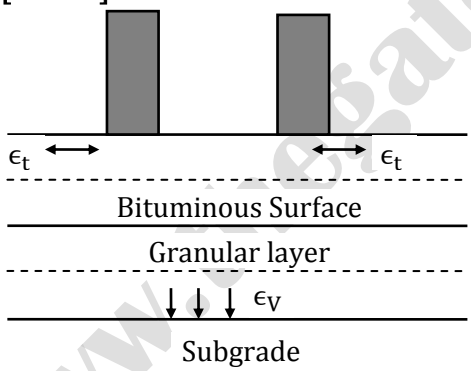
[Ans. D]



Member CD=P(Tension)
Member AB=P(Tension)
∴ Member BC=O(Consider joint B)
Member AD=O(Consider joint D)
Member AC=O
∴ 2 tension members and 3 zero force members
∴option (D) is correct

3. The initial concavity in the load-penetration curve of a CBR test is NOT due to
(A) Uneven top surface
(B) High impact at start of loading
(C) Inclined penetration plunger
(D) Soft top layer of soaked soil
[Ans. B*]
Initial concavity in CBR test due to :
- Improper compaction
- Soft top layer
- Inclined plunger

4. As per IRC:37-2012, is order to control subgrade rutting in flexible pavements , the parameter to be considered is
(A) Horizontal tensile strain at the bottom of bituminous layer
(B) Vertical compressive strain on top of subgrade
(C) Vertical compressive on top of granular layer
(D) Vertical deflection at the surface of the pavement
[Ans. B*]

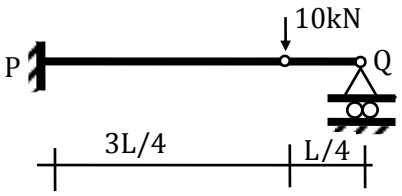


As per IRC : 37-2012

$$N = 4.1656 \times 10^{-0.08}\left[\frac{1}{\epsilon_v}\right]^{4.5337}$$

$$N = 1.41 \times 10^{-8}\left[\frac{1}{\epsilon_v}\right]^{4.5337}$$

5. A vertical load of 10kN acts on a hinge at a distance of L/4 from the roller support Q of a beam of length L



The vertical reaction at support Q is

(A) 0.0kN (B) 2.5kN

(C) 7.5kN (D) 10.0kN

[Ans. A]

Taking moment at the hinge=0

$\therefore \sum M_{Hinge} = 0$ (right side)

$\Rightarrow R_Q \times \frac{L}{4} = 0 \Rightarrow R_Q = 0$

6. As per IS 10500:2012, for drinking water in the absence of alternate source of water, the permissible limits for chloride and sulphate, in mg/L, respectively are

(A) 250 and 200 (B) 1000 and 400

(C) 200 and 250 (D) 500 and 1000

[Ans. B]

7. The setting time of cement is determined using

(A) Le chatelier apparatus (B) Briquette testing apparatus

(C) Vicat apparatus (D) Casagrande's apparatus

[Ans. C]

Vicat apparatus is used to find initial and final setting time of the cement

8. As per IS 456:2000, the minimum percentage of tension reinforcement (up to two decimal places) required in reinforced concrete beam of rectangular cross-section (Considering effective depth in the calculation of area) using Fee 500 grade steel is _____

[Ans.*] Range: 0.17 to 0.17

Minimum reinforcement in beam (in %)

$$\frac{A_{st\,min}}{bd} = \frac{0.85}{f_y} \times 100$$

$$= \frac{0.85}{f_y} \times 100$$

$$= \frac{0.85}{500} \times 100 = 0.17\%$$

9. Which one of the following statements is NOT correct?

(A) When the water content of soil lies between its liquid limit and plastic limit, the soil is said to be in plastic state.

(B) Bousinesq's theory is used for the analysis of stratified soil.

(C) The inclination of stable slope in cohesive soil can be greater than its angle of internal friction

(D) For saturated dense fine sand, after applying overburden correction, if the standard penetration test value exceeds 15, dilatancy correction is to be applied.

[Ans. B]

Bousinesq's theory is applicable for isotropic soil.

10. A culvert is designed for a flood frequency of 100 years and a useful life of 20 years. The risk involved in the design of the culvert (in percentage, up to two decimal place) is_____

[Ans. *]Range: 17.50 to 18.50*

Risk = The probability of a flood to occur at least once in n-successive years.

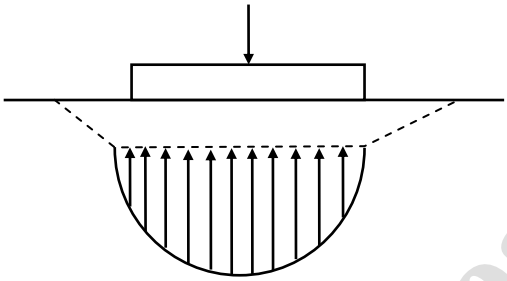
$\therefore$ Risk $= 1 - q^n$

$= 1 - (1 - P)^n$

$= 1 - (0.99)^{20}$

$= 0.18209 = 18.209\%$

11. The contact pressure and settlement distribution for a footing are shown in the figure.



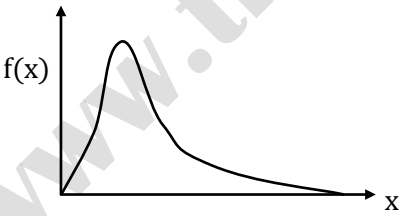
The figure corresponds to a

(A) Rigid footing on granular soil
(B) Flexible footing on granular soil
(C) Flexible footing on saturated clay
(D) Rigid footing on cohesive soil

[Ans. A]

For rigid footing settlement is uniform but contact pressure varies.

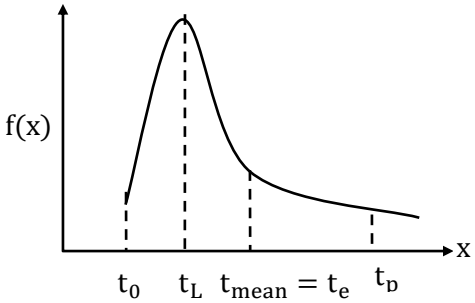
12. A probability distribution with right skew is shown in the figure.



The correct statement for the probability distribution is

(A) Mean is equal to mode
(B) Mean is greater than median but less than mode
(C) Mean is greater than median and mode
(D) Mode is greater than median

[Ans. C*]



Mode < mean

i.e., mean > median and mode

Mean is greater than the mode and the median. This is common for a distribution that is skewed to the right [i.e., bunched up toward the left and a 'tail' stretching toward the right].

13. A filet weld is simultaneously subjected to factored normal and shear stresses of 120 MPa and 50 MPa, respectively. As per IS 800: 2007 , the equivalent stress (in MPa, up to two decimal places) is _____

[Ans.*]Range: 147.50 to 148.50

$\therefore$ equivalent stress $= \sqrt{(120)^2 + 3 \times (50)^2} = 147.99$

$\simeq$ 148 MPa

14. The quadratic equation $2x^2 - 3x + 3 = 0$ is to be solved numerically starting with an initial guess as $x_0 = 2$. The new estimate of x after the first iteration using Newton-Raphson method is _____

[Ans.*] Range: 1 to 1*

$f(x) = 2x^2 - 3x + 3, x_0 = 2$

$f'(x) = 4x - 3$

By Newton-Rapshon

$$x_1 = x_0 - \frac{f(x_0)}{f'(x_0)} = 2 - \frac{2(2)^2 - 3(2) + 3}{4(2) - 3} = 2 - \frac{5}{5} = 1$$

15. A flow net below a dam consists of 24 equipotential drops and 7 flow channels. The difference between the upstream and downstream water levels is 6 m. the length of the flow line adjacent to the toe of the dam at exit is 1m. The specific gravity and void ratio of the soil below the dam are 2.70 and 0.70, respectively. The factor of safety of safety against piping is

(A) 1.67 (B) 2.5

(C) 3.4 (D) 4

[Ans. D]

$\because$ Head loss perdrop $= \frac{6}{24} = 0.25$

Hydraulic gradient $i = \frac{0.25}{1} = 0.25$

$\because$ critical hydraulic gradient $i_c = G - \frac{1}{1+e}$

$$i_c = 2.7 - \frac{1}{1+0.7} = 1$$
$$F_{os} = \frac{i_c}{i} = \frac{1}{0.25} = 4$$

16. A structural member subjected to compression, has both translation and rotation restrained at one end, while only translation is restrained at the other end. As per IS456:2000, the effective length factor recommend for design is

(A) 0.50 (B) 0.65
(C) 0.70 (D) 0.80

[Ans. D]

When one end is fixed and other end is pinned, effective length of member as per IS456:2000 is taken equal to 0.80L

17. The solution of the equation $x\frac{dy}{dx} + y = 0$ passing through the point$(1, 1)$ is

(A) x (B) x^2
(C) x^{-1} (D) x^{-2}

[Ans. C*]

$$x\frac{dy}{dx} + y = 0$$
$$x\frac{dy}{dx} = -y$$
$$\frac{dy}{y} = -\frac{dx}{x}$$
$$\int \frac{1}{y}dy = \int -\frac{1}{x}\,dx$$
$$\ln y = -\ln x + c$$
$$y = 1, x = 1$$
$$C = 0$$
$$\Rightarrow y = \frac{1}{x} = x^{-1}$$

18. The clay mineral, whose structural units are held together by potassium bond is

(A) Halloysite (B) Illite
(C) Kaolinite (D) Smectie

[Ans. B]

Structural units are held together by ionic bond or potassium bond in illite clay mineral

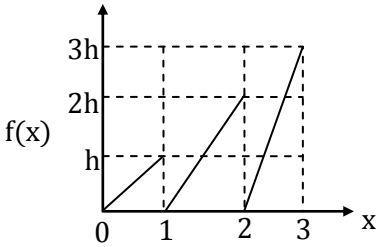
19. Dupuit's assumptions are valid for

(A) Artesian aquifer (B) Confined aquifer
(C) Leaky aquifer (D) Unconfined aquifer

[Ans. D*]

Dupuit's theory assumptions hold that groundwater flows horizontally in an unconfined aquifer and that ground water discharge is proportional to saturated aquifer thickness

20. The graph of a function f(x) is shown in the figure



For f(x) to be a valid probability density function, the value of h is

(A) 1/3 (B) 2/3

(C) 1 (D) 3

[Ans. A*]

$$\int_0^3 f(x)\,dx = 1$$

$$\int_0^1 f(x)\,dx + \int_1^2 f(x) + \int_2^3 f(x)\,dx = 1$$

$$\frac{h}{2} + \frac{h}{2} + \frac{3h}{2} = 1$$

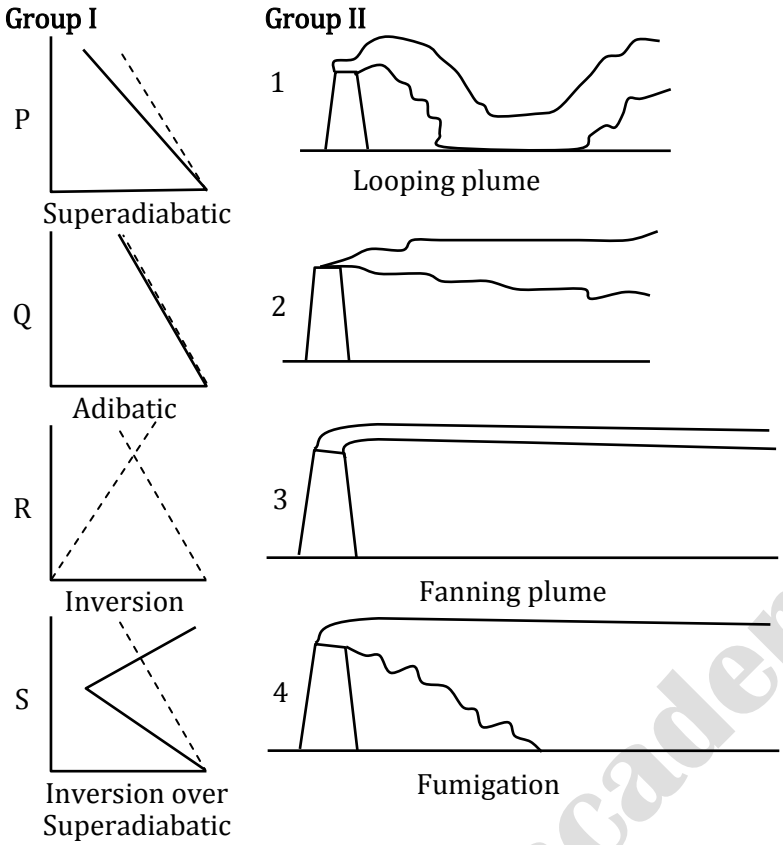
$$gh = 2 \Rightarrow h = \frac{1}{3}$$

21. Probability (up to one decimal place) of consecutively picking 3 red balls without replacement from a box continuity 5 red balls and 1 white ball is _____

[Ans. *] Range 0.5 to 0.5*

Probability $\overline{P} = \frac{5}{6} \times \frac{4}{5} \times \frac{3}{4} = \frac{1}{2} = 0.5$

22. In the figure, Group I represents the atmospheric temperature profiles (P, Q, R and S) and Group II represents dispersion of pollutants from a smoke (1, 2, 3 and 4). In the figure of Group I, the dashed line represents the dry adiabatic lapse rate, whereas the horizontal axis represents temperature and the vertical axis represents the altitude.



The correct match is

(A) P-1, Q-2, R-3, S-4
(B) P-1, Q-2, R-4, S-3
(C) P-1, Q-4, R-3, S-2
(D) P-3, Q-1, R-2, S-4

[Ans. A]

23. The intensity of irrigation for the kharif season is 50% for an irrigation project with culturable command area of 50,000 hectares. The duty for the kharif season is 1000 hectare/cumec. Assuming transmission loss of 10%, the required discharge (in cumec, up to two decimal places) at the head of the canal is _____

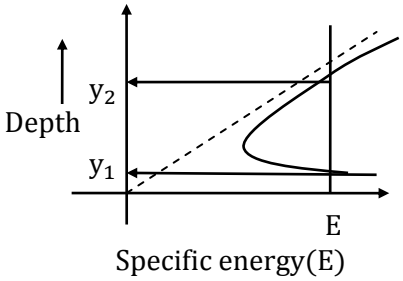
[Ans. *] Range 27.00 to 28.00*

Culturable command area= 50000 ha
Intensity of irrigation for kharif season = 50%
∴ Area under kharif= 25000 ha

24. For a given discharge in an open channel, there are two depths which the same specific energy. These two depths are known as

(A) Alternate depths
(B) Critical depths
(C) Normal depths
(D) Sequent depths

[Ans. A*]



Depth with same specific energy is called Alternate depths of flow. It represents a Subcritical depth of flow (Y_2) and a super critical depth of flow (Y_1).

25. Peak hour factor (PHF) is used to represent the proportion of peak sub-hourly traffic flow within the peak hour. If 15-minute sub-hours are considered, the theoretically possible range of PHF will be

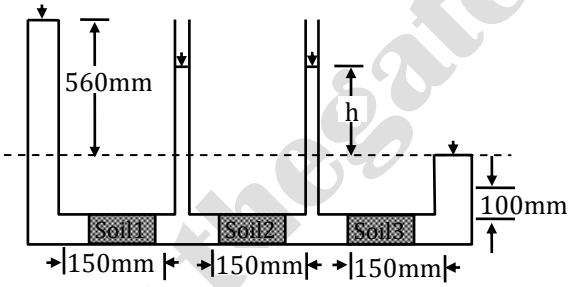
(A) 0 to 1.0
(B) 0.25 to 0.75
(C) 0.25 to 1.0
(D) 0.5 to 1.0

[Ans. C]

$0.25 \leq PHF_{15} \leq 1$

Q.26 - Q.55 Carry Two Mark each.

26. Three soil specimens (soil 1, and soil3), each 150 mm long and 100mm diameter, are placed in series in a constant head flow set-up as shown in the figure. Suitable screens are provided at the boundaries of the specimens to keep them intact. The values of coefficient of permeability of soil 1, soil2 and soil3 are 0.01, 0.003 and 0.03 cm/s, respectively.



The value of h in the set-up is

(A) 0 mm
(B) 40 mm
(C) 255 mm
(D) 560 mm

[Ans. B]

$$k_{avg} = \frac{\sum h_i}{\frac{\sum h_i}{k_i}} = \frac{150 + 150 + 150}{\frac{150}{0.01} + \frac{150}{0.03} + \frac{150}{0.03}} = 0.0064$$

$\because$ total head coss = 560 mm (Given)

$\because Q = \frac{khA}{L}$ (From constant head permeability test)

$$k_{avg} \times \frac{560}{450} \times A = k_3 \times \frac{h}{150} \times A$$

$$0.0064 \times \frac{560}{450} = 0.03 \times \frac{h}{150}$$

$h = 40$ mm

27. The total rainfall in a catchment of area 1000km^2, during a 6 h storm, is 19 cm. The surface runoff due to this storm computed from triangular direct runoff hydrograph is $1 \times 10^8 \text{m}^3$. The ϕ_{index} for this storm (in cm/h, up to one decimal place) is _____?

[Ans.*] Range: 1.50 to 1.50*

$$\text{Surface runoff} = \frac{1 \times 10^8 \text{m}^3}{1000 \times 10^6 \text{m}^2} = 0.1 \text{ m} = 10 \text{ cm}$$

Total rainfall $= 19$ cm

$$\text{Rainfall intensity} = \frac{19}{6} = 3.167 \text{ cm/hr}$$

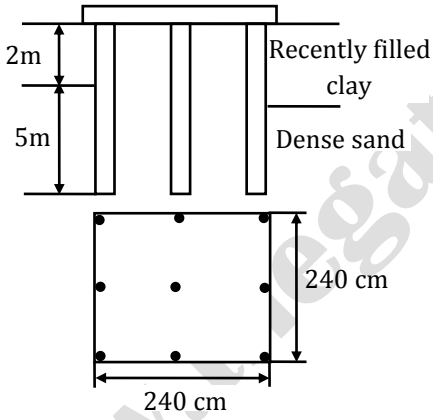
$$\text{w} - \text{index} = \frac{\text{P} - \text{Q}}{\text{t}} = \frac{\text{Total inflitartion}}{\text{Total duration of storm}}$$

$$\text{w} - \text{index} = \frac{19 - 10}{6} = 1.5 \text{ cm/hr}$$

As intensity of rainfall>w-index

And rainfall intensity is uniform therefore $\phi - \text{index} = \text{w} - \text{index} = 1.5$ cm/hr

28. A group of nine piles in a 3×3 square pattern is embedded in a soil strata comprising dense sand underlying recently filled clay layer, as shown in the figure. The perimeter of an individual pile is 126 cm. The size of pile group is 240cm × 240 cm. The recently filled clay has undrained shear strength of 15kPa and unit weight of 16kN/m^3.



The negative frictional load (in kN, up to two decimal places) acting on the pile group is

[Ans. *] Range: 472.00 to 472.50

Negative skin friction for individual pile action

$= 9 \times [0.5 \times 15 \times 1.26 \times 2]$

$= 170.1 \text{ kN/m}^2$

Negative skin frication for group of pile action

$= \propto \bar{C}\,(4BL) + \dot{\gamma}(\text{area} \times \text{length})$

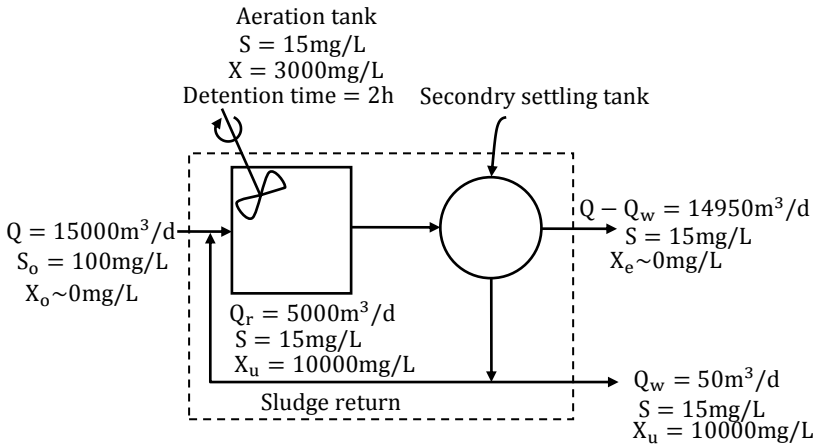
$= 1 \times 15 \times (4 \times 2.4 \times 2) + 16 \times (2.4 \times 2.4 \times 2)$

$= 472.32 \text{ kN/m}^2$

Maximum of these two will be the negative skin friction

$= 472.32 \text{ kN/m}^2$

29. A schematic flow diagram of a completely mixed biological reactor with provision for recycling of solids is shown in the figure.



So, S = readily biodegradable soluble BOD, mg/L

Q, Q_r, Q_w = flow rates, m^3/d

X_0, X, X_e, X_u = microorganism concentrations

(mixed − liquor volatile suspended solids or MLVSS), mg/L

The mean cell residence time (in days, up to one decimal place) is _____

[Ans.*] Range: 7.5 to 7.5

$$\theta_c = \frac{VX}{Q_\omega X_u}$$

$$V = Q \times 6$$

$$= \frac{15000}{24} \times 2 = 1250\ m^3$$

$$\Rightarrow X = 3000\ mg/L$$

$$X_u = 10000\ mg/L$$

$$Q_\omega = 50$$

$$\theta_c = \frac{1250 \times 3000}{50 \times 10{,}000} = 7.5\ days$$

30. The Laplace transform F(s) of the exponential function, $f(t) = e^{at}$ when $t \geq 0$, where a is a constant and $(s - a)0$, is

(A) $\frac{1}{s+a}$ (B) $\frac{1}{s-a}$

(C) $\frac{1}{a-s}$ (D) ∞

[Ans. B*]

$$L(e^{at}) = \frac{1}{s-a}$$

$$L(e^{at}) = \int_0^\infty e^{-st}\, e^{at} dt = \int_0^\infty e^{-(s-a)t} dt$$

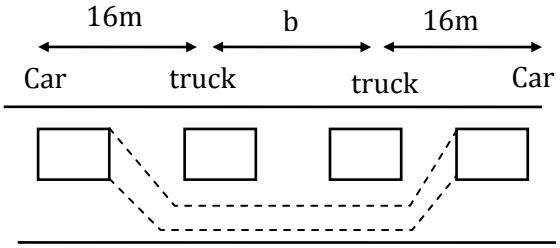
$$= \frac{e^{-(s-a)t}}{-(s-a)}\Big|_0^\infty = -\frac{1}{s-a}(0-1) = \frac{1}{s-a}$$

31. A car follows a slow moving truck (travelling at a speed of 10 m/s) on a two-lane two-way highway. The car reduces its speed to 10 m/s and follows the truck maintaining a distance of 16 m from the truck. On finding a clear gap in the opposing traffic stream, the car accelerates at an average rate of 4 m/s^2, overtakes the truck and returns to its original lane. When it returns to its original lane, the distance between the car and truck is 16 m. The total distance covered by the car during this period (from the time it leaves it lane and subsequently returns to its lane after overtaking) is

(A) 64 m
(B) 72 m
(C) 128 m
(D) 144 m

[Ans. B]

$S = ut + \frac{1}{2}at^2$



Total distance $= S = 16 + b + 16$

$S = 32 + b$

$32 + b = ut + \frac{1}{2}at^2$

u: initial speed of vehicle

b= Distance travelled by truck during overtaking time 't' with a speed of 10 m/s

$b = 10.t$

$\Rightarrow 32 + 10t = 10 \times t + \frac{1}{2} \times 4 \times t^2$

$32 + 10t = 10t + 2t^2$

$16 = t^2$

$t = 4secs$

$\Rightarrow b = 10 \times t = 10 \times 4 = 40m$

Distance $= 32 + b = 32 + 40$

$= 72m$

32. A rough pipe of 0.5 m diameter, 300 m length and roughness height of 0.25 mm, carries water (kinematic viscosity $= 0.9 \times 10^{-6} m^2/s$) with a velocity of 3 m/s. Friction factor (f) for laminar flow is given by f = 64/Re, and for turbulent flow it is given by $\frac{1}{\sqrt{f}} = 2\log_{10}\left(\frac{r}{k}\right) + 1.74$?

Where, Re is Reynolds number,

R = Radius of pipe

k = Roughness height

g = 9.81 m/s^2

The head loss (in m, up to two decimal places) in the pipe due to friction is _____?

[Ans. *] Range: 4.50 to 4.70*

$$Re = \frac{\rho.V.D}{\mu} = \frac{V.D}{v} = \frac{3 \times (0.5)}{0.9 \times 10^{-6}} = 1.67 \times 10^6$$

Means $Re > 2000$ turbulent flow

So,

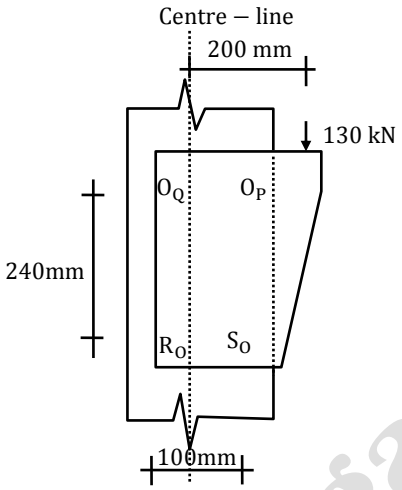
$$\frac{1}{\sqrt{f}} = 2\log_{10}\frac{D}{2k_s} + 1.74$$

$$\frac{1}{\sqrt{f}} = 2\log_{10}\frac{0.5}{2 \times 0.25 \times 10^{-3}} + 1.74$$

$$f = 0.01669$$

$$h_f = \frac{f.L.V^2}{2gD} = \frac{(0.01669)(300)(3)^2}{2 \times 9.81 \times 0.5} = 4.594 \text{ m}$$

33. Four bolts P, Q,, R and S of equal diameter are used for a bracket subjected to a load of 130 kN as shown in the figure.



The force in bolt P is

(A) 32.50 kN (B) 69.32kN

(C) 82.50 kN (D) 119.32 kN

[Ans. B]

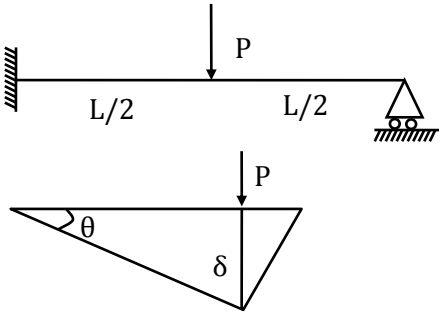
$$\therefore F_1 = \frac{130}{4} = 32.5 \text{ kN}$$

$$F_2 = \frac{130 \times 200 \times 130}{4 \times (130)^2}\left[r = \sqrt{50^2 + 120^2} = 130 \text{ mm}\right]$$

$$\therefore F_r = \sqrt{(32.5)^2 + (50)^2 + 2 \times 32.5 \times 50 \times \frac{50}{130}} = 69.34 \text{ kN}$$

34. A prismatic propped cantilever beam of span L and plastic moment capacity M_P is subjected to a concentrated load at its mid-span. If the collapse load of the beam is $\alpha\frac{M_P}{L}$, the value of α is _____

[Ans. *] Range: 6.0 to 6.0



$\therefore W_{external} = W_{internal}$

$\Rightarrow \left(\frac{1}{2}.\theta\right).P = 2M_P\theta + M_P\theta$

$\Rightarrow P = \frac{6M_P}{L}$

$\therefore \alpha = 6$

35. The total horizontal and vertical stresses at a point X in a saturated sandy medium are 170 kPa and 300 kPa, respectively. The static pore-water pressure is 30 kPa. At failure, the excess pore-water pressure is measured to be 94.50 kPa, and the shear stresses on the vertical and horizontal planes passing through the point X are zero. Effective cohesion is 0kPa and effective angle of internal friction is36°. The shear strength (in kPa, up to two decimal places) at point X is_____

[Ans. *] Range: 51.50 to 53.50

$\because \tau = C + \bar{\sigma}_n \tan\phi$

$\sigma_n = \frac{300+170}{2} + \frac{300-170}{2}\cos 2\left(45 + \frac{36}{2}\right)$

$\sigma_n = \frac{300+170}{2} + \frac{300-170}{2}\cos 126°$

$\sigma_n = 196.79\ kP_a$

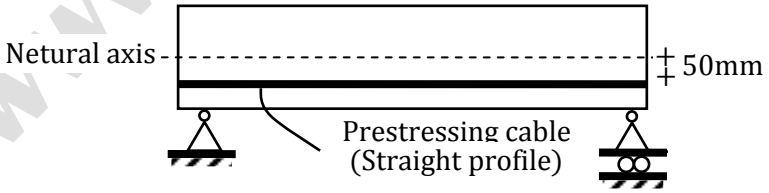
$\bar{\sigma}_n = \sigma_n - u = 196.79 - (30 + 94.5)$

$\bar{\sigma}_n = 72.29\ kP_a$

$\therefore \tau = 72.29 \tan 36°$

$\tau = 52.52$ kPa

36. A 6 m long simply-supported beam is Pre-stressed as shown in the figure.



The beam carries a uniformly distributed load of 6 kN/m over it entire span. If the effective flexural rigidity $EI = 2 \times 10^4\ kN/m^2$ and the effective Prestressing force is 200 kN, the net increase in length of Pre-stressing cable (in mm, up to two decimal places) is _____?

[Ans. *] Range: 0.10 to 0.12 *

Span of PSC beam=6m

$EI = 2 \times 10^4 kNm^2 = 2 \times 10^4 \times 1000^3\ N - mm^2$

$= 2 \times 10^{13}\text{N} - \text{mm}^2$

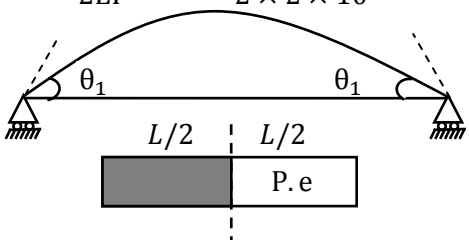
$P = 200\ \text{kN}$

Total UDL=6 kN/m

Eccentricity $= e = 50\ \text{mm}$

(a) Slope of beam due to P-force

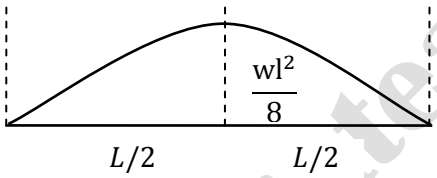
$$\theta_1 = \frac{P.eL}{2EI} = \frac{200 \times 10^3 \times 50 \times 6000}{2 \times 2 \times 10^{13}} = 1.5 \times 10^{-3}(\text{upward})$$



BMD due to prestressing

(b) Slope of beam due to UDL

$$\theta_2 = \frac{wL^3}{24EI} = \frac{6 \times (6000)^3}{24 \times 2 \times 10^{13}} = (+)2.7 \times 10^{-3}(\text{downward})$$



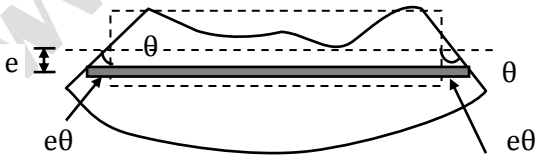
BMD due to loading

(c) Net slope of beam

$\theta = \theta_1 + \theta_2$

$= -1.5 \times 10^{-3}(+)2.7 \times 10^{-3} = 1.2 \times 10^{-3}$

(d) Total net increase in length



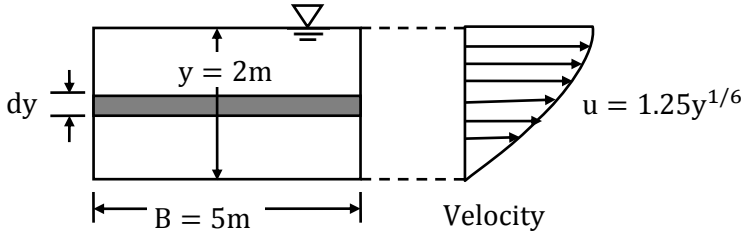
$2e\theta = 2 \times 50 \times 1.2 \times 10^{-3} = 0.12\ \text{mm}$

37. In a 5m wide rectangular channel, the velocity u distribution in the vertical direction y is given by $u = 1.25\, y^{\frac{1}{6}}$. The distance y is measured from the channel bed. If the flow depth is 2m, the discharge per unit width of the channel is

(A) $2.40\ \text{m}^3/\text{s/m}$ (B) $2.80\ \text{m}^3/\text{s/m}$

(C) $3.27\ \text{m}^3/\text{s/m}$ (D) $12.02\ \text{m}^3/\text{s/m}$

[Ans. A*]



Discharge through elementary strip (dQ) = 5dy × u

Total discharge, $Q = \int_0^2 (5dy)\left(1.25y^{\frac{1}{6}}\right)$

$$\Rightarrow Q = 6.25\int_0^2 y^{\frac{1}{6}}dy = 6.25 \times \left[\frac{y^{\frac{1}{6}+1}}{\frac{1}{6}+1}\right]_0^2$$

$Q = 12.026\ m^3$

$\therefore$ Dicharge per unit width, $q = \frac{12.026}{5} = 2.405\frac{m^3}{s}/m$

38. The rank of the following matrix is

$$\begin{bmatrix}1 & 1 & 0 & -2\\ 2 & 0 & 2 & 2\\ 4 & 1 & 3 & 1\end{bmatrix}$$

(A) 1 (B) 2

(C) 3 (D) 4

[Ans. B*]

$$A = \begin{bmatrix}1 & 1 & 0 & -2\\ 2 & 0 & 2 & 2\\ 4 & 1 & 3 & 1\end{bmatrix}$$

$R_2 \rightarrow R_2 - 2R_1, R_3 \rightarrow R_3 - 4R_1$

$$A = \begin{bmatrix}1 & 1 & 0 & -2\\ 0 & -2 & 2 & 6\\ 0 & -3 & 3 & 9\end{bmatrix}$$

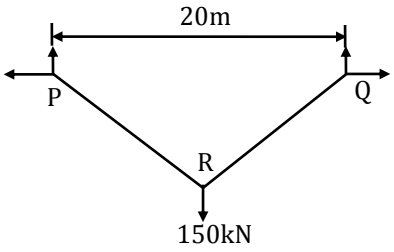
$R_3 \rightarrow R_3 - \frac{3}{2}R_2$

$$A = \begin{bmatrix}1 & 1 & 0 & -2\\ 0 & -2 & 2 & 6\\ 0 & 0 & 0 & 0\end{bmatrix}$$

No. of non- zero rows = 2

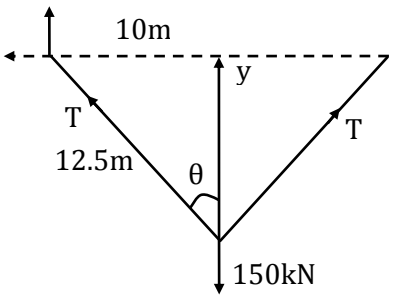
Rank of A = 2

39. A cable PQ of length 25 m is supported at two ends at the same level as shown in the figure. The horizontal distance between the supports is 20m. A point load of 150 kN is applied at point R which divides it into two equal parts.



Neglecting the self-weight of the cable, the tension (in kN, in integer value) in the cable due to the applied load will be _____

[Ans. *] Range: 125 to 125



$y^2 + 10^2 = (12.5)^2 \Rightarrow y = 7.5\text{m}$

$\therefore T.\cos\theta + T.\cos\theta = 150$

$\Rightarrow 2T.\cos\theta = 150$

$\Rightarrow 2 \times T \times \frac{7.5}{12.5} = 150$

$\Rightarrow T = 75 \times \frac{5}{3} \Rightarrow T = 125 \text{ kn}$

40. At a small water treatment plant which has 4 filters, the rates of filtration and back washing are 200 $m^3/d/m^2$ and 1000 $m^3/d/m^2$, respectively. Backwashing is done for 15 minutes/day. The maturation, which occurs initially as the filter is put back into service after cleaning, takes 30min. It is proposed to recover the water being wasted during back washing and maturation. The percentage increase in filtered water produced (up to two decimal places) would be _____

[Ans. *] Range: 7.75 to 7.95

Assume the area $= 1\ m^2$

Volume of water during backwashing

$= 1000 \times \frac{15}{24 \times 60} \times 1 = 10.41\ m^3$

Volume of water during maturation

$= 200 \times \frac{30}{24 \times 60} = 4.166 m^3$

Total volume $= 10.41 + 4.166 = 14.58 m^3$

Duration for which filter is operational

$= 23$ hrs 15 mins

$= 23.25$ hrs

$\Rightarrow$ In 23.25 hrs water filtered $= (200 \times 1) \times \frac{23.25}{24} = 193.75\ m^3/\text{day}$

$$\% \text{ inc in filtered water} = \frac{14.58}{193.75} \times 100$$
$$= 7.52\%$$

41. The matrix $\begin{bmatrix} 2 & -4 \\ 4 & -2 \end{bmatrix}$ has

(A) Real eigenvalues and Eigen vectors
(B) Real eigenvalues but complex Eigen vectors
(C) Complex eigenvalues but real Eigen vectors
(D) Complex eigenvalues and Eigen vectors

[Ans. D*]

$A = \begin{bmatrix} 2 & -4 \\ 4 & -2 \end{bmatrix}$

$|A - \lambda I| = 0$

$\begin{vmatrix} 2-\lambda & -4 \\ 4 & -2-\lambda \end{vmatrix} = 0$

$-4 - 2\lambda + 2\lambda + \lambda^2 + 16 = 0$

$\lambda^2 + 12 = 0$

$\lambda = \pm 2\sqrt{3}i$

$\lambda = 2\sqrt{3}i$

Consider $(A - \lambda I)X = 0$

$\begin{bmatrix} 2-2\sqrt{3}i & -4 \\ 4 & -2-2\sqrt{3}i \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \end{bmatrix}$

$2 - 2\sqrt{3}i_{x1} = 4x_2$

$\frac{x_1}{4} = \frac{x_2}{2-2\sqrt{3}i}$

$\begin{bmatrix} x_1 \\ x_2 \end{bmatrix} = \begin{bmatrix} 4 \\ 2-2\sqrt{3}i \end{bmatrix}$

$\lambda = -2\sqrt{3}i$

Consider $(A - \lambda I)X = 0$

$\begin{bmatrix} 2+2\sqrt{3}i & -4 \\ 4 & -2+2\sqrt{3}i \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \end{bmatrix}$

$2 + 2\sqrt{3}ix_1 = 4x_2$

$\frac{x_1}{4} = \frac{x_2}{2+2\sqrt{3}i}$

$\begin{bmatrix} x_1 \\ x_2 \end{bmatrix} = \begin{bmatrix} 4 \\ 2+2\sqrt{3}i \end{bmatrix}$

$\therefore$ Complex eigen values and complex eigen vectors

42. A coal containing 2% sulfur is burned completely to ash in a brick kiln at a rate of 30 kg/min. The sulfur content in the ash was found to be 6% of the initial amount of sulfur present in the coal fed to the brick kiln. The molecular weight of S, H and o are 32, 1 and 16 g/mole, respectively. The annual rate of sulfur dioxide (SO_2) emission from the kiln (in tonnes/year, up to two decimal places) is _____

[Ans. *] Range: 590.00 to 595.00

Rate → 30 kg/min

In one year, total amount of coal which is required $= 30 \times 24 \times 60 \times 365$

$= 15768000$kg

Sulphur content $\rightarrow 2\% = \frac{2}{100} \times 15768000$

$= 315{,}360$ kg

$= 315.36$ tonnes/year

6% of $315.36 \rightarrow 0.06 \times 315.36 = 18.92$ tonnes/year

Remaining sulphur is converted to SO_2

$315.36 - 18.92 = 296.44$ tonnes/year

$S + O_2 \rightarrow SO_2$

32gm → 64 gm

1 gm → 2 gm

$296.44 \rightarrow 2 \times 296.44$

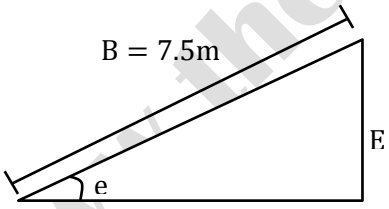
tonnes = 592.88 tonnes

43. A 7.5 m wide two-lane road on a plain terrain is to be laid along a horizontal curve of radius 510m. For a design speed of 100 kmph, super-elevation is provided as per IRC: 73-1980. Consider acceleration due to gravity as 9.81 m/s^2. The level difference between the inner and outer edges of the road(in. m, up to three decimal places) is _____

[Ans. *] Range: 0.520 to 0.530

Design speed = 100 kmph

Radius = 510 m



Design super elevation $= \frac{V^2}{225\ R}$

$e_{design} = 0.087$

$\Rightarrow e_{design} > e_{max}$

provide $e = 0.07$

$\Rightarrow e = \tan\theta = \sin\theta = \frac{E}{B}$

(small angle $\theta \rightarrow \tan\theta = \sin\theta$)

$0.07 = \frac{E}{7.5}$

$0.525\ M = E$

44. A single –reinforced rectangular concrete beam of width 300 mm and effective depth 400 mm is to be designed using M25 grade concrete and F_e500 grade reinforcing steel. For the beam to be under-reinforced, the maximum number of 16mm diameter reainforcing bars that can be provided is

(A) 3 (B) 4

(C) 5 (D) 6

[Ans. C]

Given data $p = 300$ mm

$d = 400$ mm

$f_{ck} = 25\ N/mm^2$

For under-reinforced section $X_u < X_{u,max}$

$\because X_{u\,max} = 0.46\,d(\text{For Fe500}) = 0.46 \times 400$

$X_{u\,max} = 184$ mm

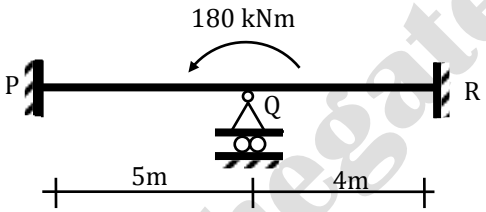
$$\because X_u = \frac{0.87\,f_y\left[n\frac{\pi}{4}(16)^2\right]}{0.36\,f_{ck}b}$$

$$\because \frac{0.87 \times 500 \times n\,(201)}{0.36 \times 25 \times 30} < 184$$

$n < 5.68$

Maximum number of 16 mm diameter bars that can be provided in under reinforced beam are 5.

45. A prismatic beam P-Q-R of flexural rigidity $EI = 1 \times 10^4 kNm^2$ is subjected to a moment of 180 kNm at Q as shown in figure .



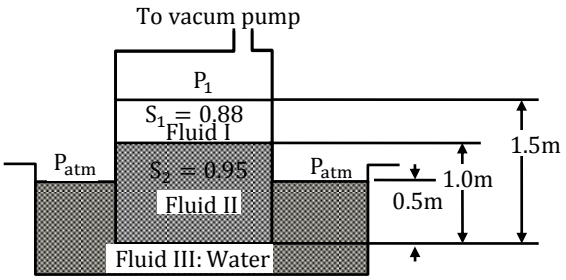
The rotation at Q (in rad, up to two decimal places) is _____

[Ans. *] Range: 0.01 to 0.01

$$\frac{4EI}{5}(\theta_Q) + \frac{4EI}{4}(\theta_Q) = 180\ kN - m$$

$\Rightarrow \theta_Q = 0.01$ radian

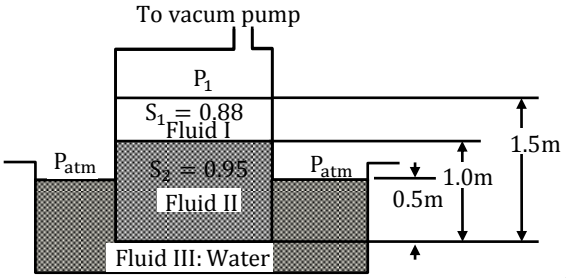
46. A three fluid system (immiscible) is connected to a vacuum pump. The special gravity values of the fluids (S_1, S_2) are given in figure.The guage pressure value $\left(\text{in}\frac{\text{kN}}{\text{m}^2}\right)$,upto two



unit weight of water, $\gamma_w = 9.81\ \text{kNm}^3$
Atmospheric Pressure, $P_{atm} = 95.43$ kPa

The gauge pressure value (in kN/m^2, up to two decimal places)of P_1 is _____

[Ans. *] Range: -9.0 to -8.0*



Taking P_1 is in gauge pressure.

$$P_A = P_1 + (0.88 \times 103) \cdot (9.81)(0.5) + (0.95 \times 10^3)(9.81)(1)$$

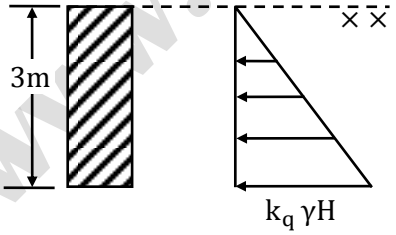
$$(10^3)(9.81)(0.5) = P_1 + (0.88 \times 10^3) \cdot (9.81)(0.5) + (0.95 \times 10^3)(9.81)(1)$$

$$P1 = -8.73\ \text{kN/m}^2$$

47. A 3m high vertical earth retaining wall retains a dry granular backfill with angle of internal friction of 30° and unit weight of $20\ \text{kN/m}^3$. If the wall is prevented from yielding (no movement), the total horizontal thrust (in kN per unit length) on the wall is

(A) 0 (B) 30
(C) 45 (D) 270

[Ans. C]



$$K_o = 1 \sin \phi = 1 - \sin 30^\circ = 0.5$$

$$P_o = \frac{1}{2} K_o V H^2$$

$$= \frac{1}{2} \times 0.5 \times 20 \times (30)^2$$

$$= 45\ \text{kN/m}$$

48. A flocculation tank contains $1800 m^3$ of water which is mixed using paddles at an average velocity gradient G of 100/s .The water temperature and the corresponding dynamic viscosity are 30°C and 0.798×10^{-3} Ns/m^2 respectively. The theoretical power required to achieve the stated value of G (in kW, up to two decimal place) is _____

[Ans. *] Range: 14.00 to 15.00

$G = 100\,sec^{-1}$

$\mu = 0.798 \times 10^{-3}\ N - s/m^2$

$V = 1800\ m^3$

power required

$$\Rightarrow G = \sqrt{\frac{P}{\mu v}}$$

$G^2 . \mu v = P$

$100^2 \times 0.798 \times 10^{-3} \times 1800 = P$

$= 14.634\ k\omega = P$

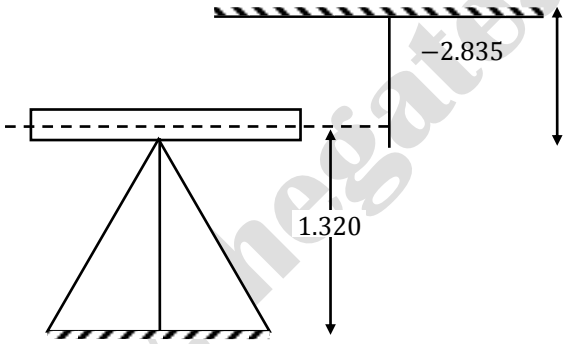
49. A level instrument at a height of 1.320 m has been placed at a station having a reduced level (RL) of 112.565 m. The instrument reads −2.835 m on a levelling staff held at the bottom of a bridge deck. The RL(in m) of the bottom of the bridge deck is

(A) 116.720 (B) 116.080

(C) 114.080 (D) 111.050

[Ans. A]

R.L of bottom of bridge deck



R. L = 112.565 m

$= 112.565$

$+$

1.320

$+$

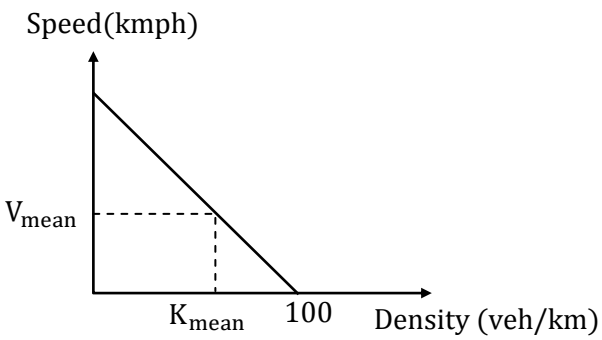
2.835

$= 116.720$ m

50. The space mean speed (kmph) and density (vehicles/km) of a traffic stream are linearly related. The free flow speed and jam density are 80 kmph and 100 vehicles/km respectively. The traffic flow (in vehicles/h, up to one decimal place) corresponding to a speed of 40 kmph is _____

[Ans.*] Range: 2000.0 to 2000.0

$V_f = 80$ kmph
$k_j = 100$ veh/km



$\because$ It is a linear variation

$\Rightarrow V_{mean} = \frac{80+0}{2} = 40$ kmph

and at V_{mean}, density is k_{mean}

$\Rightarrow k_{mean} = \frac{100+0}{2} = 50$ veh/km

At $V = 40$ kmph $\rightarrow k = 50$ veh/km

$\Rightarrow q = k.v$

$= 40 \times 50 = 2000$ veh/hr

51. An aerial photograph of terrain having an average elevation of 1400 m is taken at a scale of 1:7500. The focal length of the camera is 15cm. The altitude of the flight above mean sea level (in m, up to one decimal place) is _____

[Ans. *] Range: 2520.0 to 2530.0

$h = 1400$ m

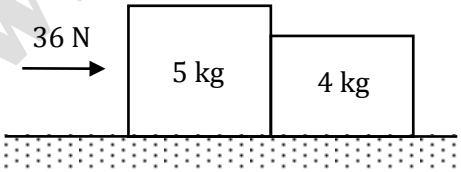
$s = 1:75000$

$s = \frac{f}{H-h}$

$\frac{1}{7500} = \frac{15 \times 10^{-2}}{H-1400}$

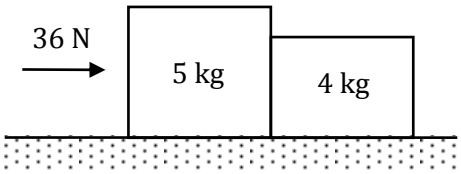
$H = 2525$m

52. Two rigid bodies of mass 5 kg and 4 kg are at rest on a frictionless surface until acted upon by a force of 36 N as shown in figure. The contact force generated between the two bodies is

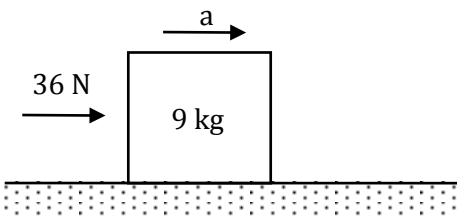


(A) 4.0 N
(B) 7.2 N
(C) 9.0 N
(D) 16.0 N

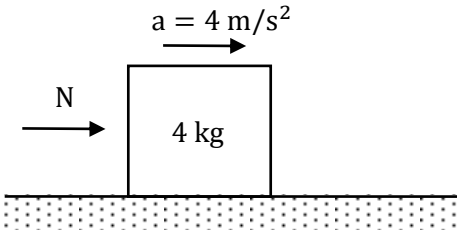
[Ans. D*]



As after action of 36 N, both blocks will move with same acceleration so considering 5 kg and 4 kg together in a system and applying Newton's 2nd law.



$a = \frac{36}{9} = 4\ m/s^2$

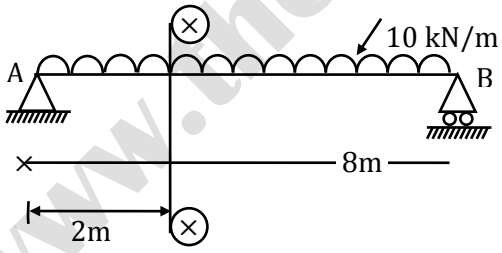


N = ma {Newton's 2nd law}

$N = 4 \times 4 = 16\ N$

53. An 8 m long simply –supported elastic beam of rectangular cross-section (100 mm × 200 mm) is subjected to a uniformly distributed load of 10 kN/m over its entire span. The maximum principle stress (in MPa, up to two decimal places)at a point located at the extreme compression edge of a cross-section and at 2 m from the support is _____

[Ans. *]Range: 0.0 to 0.0

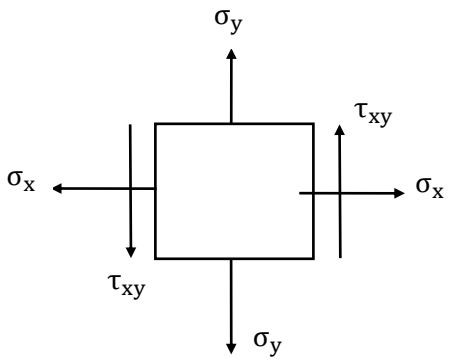


$\therefore M_X = R_A\, x - \frac{(w.x)x}{2} = (40 \times 2) - (10 \times 2) \times \frac{2}{2}$

$= 60\ kN - m$

we know, $\frac{M}{I} = \frac{\sigma}{y}$ (σ = bending stress)

$\Rightarrow \sigma = \frac{M}{z} = 60 \times \frac{10^6}{100 \times \frac{(200)^2}{6}} = 90\ N/mm^2$



$\sigma_x = -90$
$\sigma_y = 0$
$\tau_{xy} = 0$
$\sigma_1 = 0$
$\sigma_2 = -90$
Hence 0 MPa

54. The compression curve (void ratio, e vs. effective stress, σ'_v) for a certain clayey soil is a straight line in a semi –logarithmic plot and it passes through the points ($e = 1.2\,; \sigma_v = 50$ kPa)and ($e = 0.6; \sigma'_v = 800$ kPa). The compression index(up to two decimal places) of the soil is _____

[Ans. *] Range: 0.45 to 0.55

$$C_c = \frac{\Delta e}{\log\left(\frac{\bar{\sigma}_2}{\bar{\sigma}_1}\right)} = \frac{1.2 - 0.6}{\log\left(\frac{800}{50}\right)}$$

$$C_c = \frac{0.6}{\log(16)} = 0.4983$$

55. The value (up to two decimal places) of a line integral $\int_C \vec{F}(\vec{r}).d\vec{r}$, for $\vec{F}(\vec{r}) = x^2\hat{i} + y^2\hat{j}$ along C which is a straight line joining (0, 0) to (1, 1) is _____

[Ans. *] Range: 0.60 to 0.70*

$\bar{F} = x^2\bar{i} + y^2\bar{j}$

$\int \bar{F}.d\bar{r} = \int (x^2\bar{i} + y^2\bar{j})(dx\bar{i} + dy\bar{j})$

$= \int x^2dx + y^2dy$

(0,0) to (1,1) line is y=x

$$= \int x^2dx + x^2dx = \int_0^1 2x^2\,dx$$

$$= 2\left(\frac{x^3}{3}\right)\Big|_0^1 = \frac{2}{3} = 0.666$$