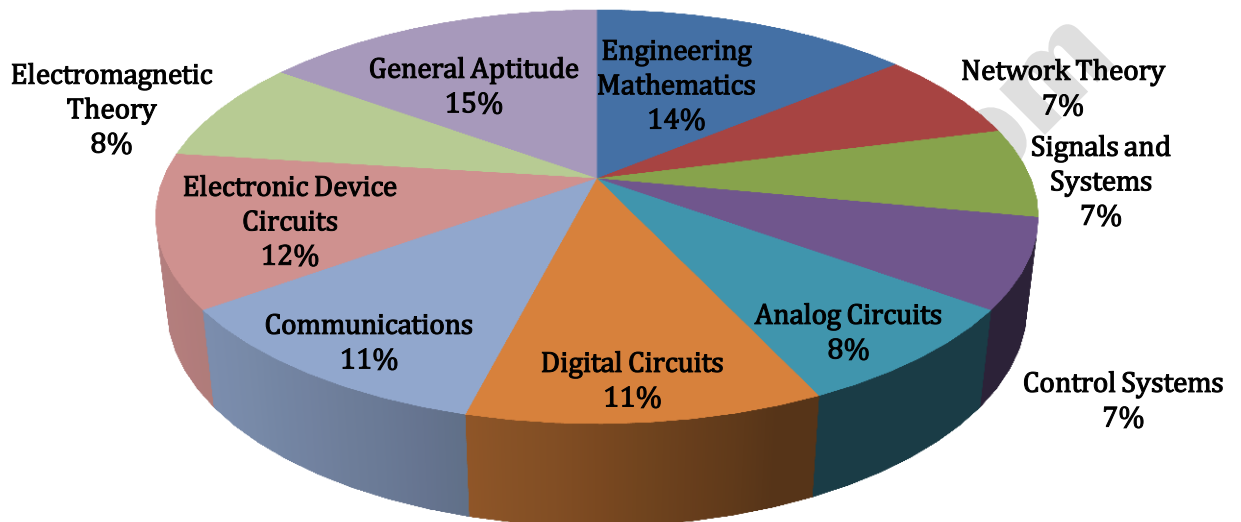


## ANALYSIS OF GATE 2018

### Electronics and Communication Engineering



**ECE ANALYSIS-2018\_10-Feb\_Morning**

SUBJECT	No. of Ques.	Topics Asked in Paper(Memory Based)	Level of Ques.	Total Marks
Engineering Mathematics	1 Marks: 6 2 Marks: 4	Random Variables; Partial Derivative; Taylor's Series, polynomials, Eigen Values, Residue Differential Equation	Moderate	14
Network Theory	1 Marks: 1 2 Marks: 3	Basic Components and types of circuits; RC Circuits; RLC Circuits Two Port Networks	Easy	7
Signals and Systems	1 Marks: 3 2 Marks: 2	Linearity, Pole-zero NQ camp, FS, DFT	Easy	7
Control Systems	1 Marks: 1 2 Marks: 3	Feedback Controllers; State Space, Bode plot TD Analysis	Moderate	7
Analog Circuits	1 Marks: 2 2 Marks: 3	Operational Amplifiers ; Trans Imp Amplifier; Small Signal AM Diode; Zener Diode	Easy	8
Digital Circuits	1 Marks: 3 2 Marks: 4	FSM, CMOS, LF, MUX, FF, ROM	Moderate	11
Communications	1 Marks: 3 2 Marks: 4	AM, Binary Channel; Probability of error, Gaussian Noise HT, RV	Tough	11
Electronic Device Circuits	1 Marks: 4 2 Marks: 4	EB; Built in Potential; NMOS, CMOS, IC Fabrications; P-N Junction, Solar cell	Easy	12
Electromagnetic Theory	1 Marks: 2 2 Marks: 3	SMIT, Transmission Lines, WG, OI, SD	Easy	8
General Aptitude	1 Marks: 5 2 Marks: 5	Filling Blanks, Geometry Numbers, GP, Mixt, CI, Probability	Easy	15
<b>Total</b>	<b>65</b>			<b>100</b>
<b>Faculty Feedback</b>	Overall paper is tougher compared to last year. Mathematics is moderate level and aptitude is easy.			

## GATE 2018 Examination

### Electronics and Communication Engineering

Test Date: 10-Feb-2018

Test Time: 9:00 AM 12:00 PM

Subject Name: Electronics and Communication Engineering

#### General Aptitude

**Q.1 - Q.5 Carry One Mark each.**

1. "By giving him the last \_\_\_\_\_ of cake, you will ensure lasting \_\_\_\_\_ in our house today"
- (A) Peas, Piece (B) Piece, Peace  
(C) Peace, Piece (D) Peace, Peas

**[Ans. B\*]**

- Peace is a situation when there is no war or fighting and things are calm and quiet.
- Piece is a part of something.

2. Even though there is a vast scope for its \_\_\_\_\_, tourism has remained a/an \_\_\_\_\_ area. The words that best fill the blanks in the above sentence are
- (A) Improvement, Neglected (B) Rejection, Approved  
(C) Fame, Glam (D) Interest, Disinterested

**[Ans. A\*]**

Even though there is a vast scope for its improvement, tourism has remained a neglected area.

3. If the number 715■423 is divisible by 3 (■ denotes the missing digit in the thousandths place), then the smallest whole number in the place of ■ is\_\_\_\_\_.
- (A) 0 (B) 2  
(C) 5 (D) 6

**[Ans. B\*]**

715 ? 423

We know that the divisibility rule for 3 is sum of all digits should be divisible by 3.

$$7 + 1 + 5 + 4 + 2 + 3 = 22$$

So, the next number after sum 22 which are divisible by 3 are 24, 27, 30 etc.

$$\text{So, } 22 + 2 = 24; 22 + 5 = 27$$

But according to minimum condition 2 is right answer.

4. What is the value of  $1 + \frac{1}{4} + \frac{1}{16} + \frac{1}{64} + \frac{1}{256} + \dots$

- (A) 2 (B) 7/4  
(C) 3/2 (D) 4/3

**[Ans. D\*]**

This is a infinite Geometric progression with first term (a) = 1

And common ratio (r) =  $\frac{1}{4}$

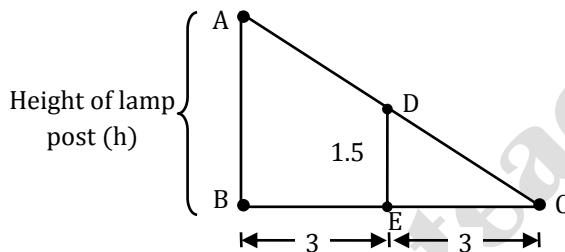
The formula for finding the sum of infinite G. P is given by  $\frac{a}{1-r}$

$$\frac{1}{1 - \frac{1}{4}} = \frac{1}{3/4} = \frac{4}{3}$$

5. A 1.5 m tall person is standing at a distance of 3 m from a lamp post. The light from the lamp at the top of the post casts her shadow. The length of the shadow is twice her height. What is the height of the lamp post in meters?

- (A) 1.5 (B) 3  
(C) 4.5 (D) 6

**[Ans. B\*]**



ABC and DEC are similar triangles

$$\frac{AB}{DE} = \frac{BC}{EC}$$

$$\frac{h}{1.5} = \frac{6}{3}$$

$$\frac{h}{1.5} = 2$$

$$h = 3 \text{ meter}$$

**Q.6 - Q.10 Carry Two Mark each.**

6. A cab was involved in a hit and run accident at night. You are given the following data about the cabs in the city and the accident.

- (i) 85% of cabs in the city are green and the remaining cabs are blue.  
(ii) A witness identified the cab involved in the accident as blue.  
(iii) It is known that a witness can correctly identify the cab colour only 80% of the time.

Which of the following options is closest to the probability that the accident was caused by a blue cab'?

- (A) 12% (B) 15%  
(C) 41% (D) 80%

**[Ans. C\*]**

Let us suppose total numbers of cabs are 100.

85 are given and rest 15 is blue.

Witness is correct 80% of times in identifying.

Total number of blue cabs identified correctly.

$$(80\% \text{ of } 15) \Rightarrow \frac{80}{100} \times 15 = 12$$

Witness is incorrect 20% of times

That incorrectness should happen for green

$$(20\% \text{ of } 85) \Rightarrow \frac{20}{100} \times 85 = 17$$

Witness identifies  $12 + 17 = 29$  Cabs in total.

$$\text{Probability} = \frac{\text{Required}}{\text{Total}} = \frac{12}{12 + 17} = \frac{12}{29} = 41.3\% \approx 41\%$$

7. Leila aspires to buy a car worth Rs. 10,00,000 after 5 years. What is the minimum amount in Rupees that she should deposit now in a bank which offers 10% annual rate of interest, if the interest was compounded annually?

(A) 5,00,000

(B) 6,21,000

(C) 6,66,667

(D) 7,50,000

[Ans. B\*]

$$A = P \left( 1 + \frac{r}{100} \right)^n$$

$$1000000 = P \left( 1 + \frac{10}{100} \right)^5$$

$$1000000 = P \left( \frac{11}{10} \right)^5$$

$$1000000 = P \left( \frac{11^5}{10^5} \right)$$

$$P = 1000000 \times \frac{10^5}{11^5} = \frac{100^{11}}{11^5} = 620921.323 = 621000$$

8. The Cricket Board has long recognized John's potential as a leader of the team. However, his on-field temper has always been a matter of concern for them since his junior days. While this aggression has filled stadia with die-hard fans, it has taken a toll on his own batting. Until recently, it appeared that he found it difficult to convert his aggression into big scores. Over the past three seasons though, that picture of John has been replaced by cerebral, calculative and successful bats man-captain. After many years, it appears that the team has finally found a complete captain.

Which of the following statements can be logically inferred from the above paragraph?

(i) Even as a junior cricketer, John was considered a good captain.

(ii) Finding a complete captain is a challenge.

(iii) Fans and the flicker Board have differing views on what they want in a captain

(iv) Over the past three seasons John has accumulated big scores.

(A) (i), (ii) and (iii) only

(B) (iii) and (iv) only

(C) (ii) and (iv) only

(D) (i), (ii), (iii) and (iv)

**[Ans. C\*]**

Statement (i) is not true as nowhere it is mentioned that John was a captain in junior team. The introductory line emphasizes on the board recognizing John's potential (Latent quality/possibility) as leader of the team.

Statement (iii) also manipulates the facts mentioned in the argument.

The 3rd statement of the argument while this aggression has filled stadia with die-hard fans does not indicate fans expectations from John as a captain.

Statement (ii) The concluding statement of the para suggests that finding a complete captain is a tough task as it took John many years to become a successful and calculative batsman - captain.

Statement (iv) can be explicitly concluded from the last 4 lines of the para.

9. Two alloys A and B contain gold and copper in the ratios of 2:3 and 3:7 by mass. respectively. Equal masses of alloys A and B are melted to make an alloy C. The ratio of gold to copper in alloy C is\_\_\_\_\_.

(A) 5:10

(B) 7:13

(C) 6:11

(D) 9:13

**[Ans. B\*]**

Alloy A contains Gold and Copper.

Let a : 3x

Which is same as 4x : 6x Alloy B contains Copper 3x : 7x

As masses of Alloy A is equal to Alloy of mass B. i.e.

	Gold	Copper	
i.e., Alloy 1	4x :	6x	⇒ 10x (mass)
Alloy 1	3x :	7x	⇒ 10x (mass)

Ratio of Gold to Copper is,

$$\frac{\text{Gold from alloy 1} + \text{Gold of alloy 2}}{\text{Copper from alloy 1} + \text{Copper of alloy 2}} = \frac{4x + 3x}{6x + 7x} = \frac{7x}{13x} = \frac{7}{13}$$

10. A coastal region with unparalleled beauty is home to many species of animals. It is dotted with coral reefs and unspoilt white sandy beaches. It has remained inaccessible to tourists due to poor connectivity and lack of accommodation. A company has spotted the opportunity and is planning to develop a luxury resort with helicopter service to the nearest major city airport. Environmentalists are upset that this would lead to the region becoming crowded and polluted like any other major beach resorts.

Which one of the following statements can be logically inferred from the information given in the above paragraph?

(A) The culture and tradition of the local people will be influenced by the tourists.

(B) The region will become crowded and polluted due to tourism.

(C) The coral reefs are on the decline and could soon vanish.

(D) Helicopter connectivity would lead to an increase in. tourists coming to the region.

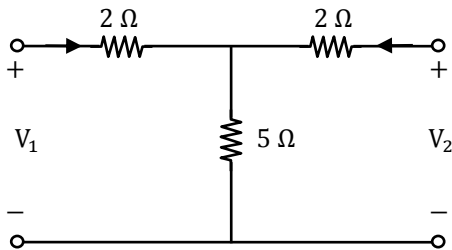
**[Ans. D]**

**Technical**

Q.1 - Q.25 Carry One Mark each.

1. The ABCD matrix for a two-port network is defined by:

$$\begin{bmatrix} V_1 \\ I_1 \end{bmatrix} = \begin{bmatrix} A & B \\ C & D \end{bmatrix} \begin{bmatrix} V_2 \\ -I_2 \end{bmatrix}$$

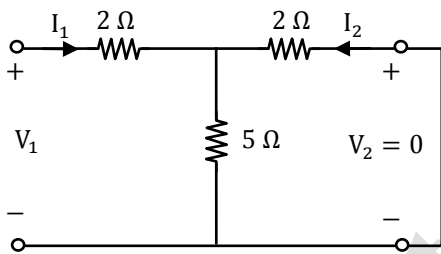


The parameter B for the given two-port network (in ohms, correct to two decimal places) is \_\_\_\_\_.

[Ans. \*] Range: 4.3 to 5.3\*

$$B = \left. \frac{V_1}{I_2} \right|_{V_2=0}$$

When  $V_2 = 0$  (i.e., when port-2 is short circuited)

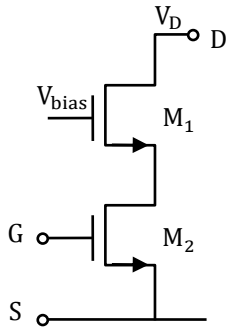


$$I_1 = \frac{V_1}{2\Omega + (5\Omega \parallel 2\Omega)} = \frac{7V_1}{24\Omega}$$

$$I_2 = -I_1 \times \frac{5\Omega}{5\Omega + 2\Omega} = \frac{-5V_1}{24\Omega}$$

So,  $B = -\frac{V_1}{I_2} = \frac{24}{5} \Omega = 4.80 \Omega$

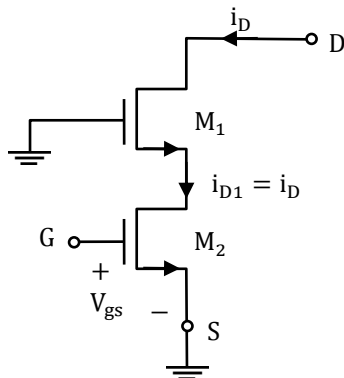
2. Two identical nMOS transistors  $M_1$  and  $M_2$  are connected as shown below. The circuit is used as an amplifier with the input connected between G and S terminals and the output taken between D and S terminals.  $V_{bias}$  and  $V_D$  are so adjusted that both transistors are in saturation. The trans conductance of this combination is defined as  $g_m = \frac{\partial i_D}{\partial v_{GS}}$  while the output resistance is  $r_0 = \frac{\partial v_{DS}}{\partial i_D}$ , where  $i_D$  current flowing into the drain of  $M_2$ . Let  $g_{m1}, g_{m2}$  be the trans conductances and  $r_{o1}, r_{o2}$  be the output resistances of transistors  $M_1$  and  $M_2$  respectively.



Which of the following statements about estimates for  $g_m$  and  $r_o$  is correct?

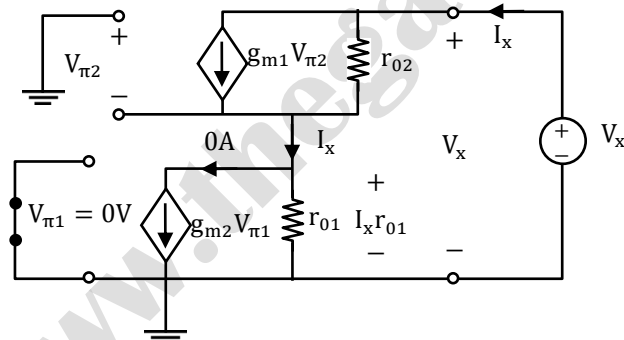
- (A)  $g_m \approx g_{m1} \cdot g_{m2} \cdot r_{o2}$  and  $r_o \approx r_{o1} + r_{o2}$       (B)  $g_m \approx g_{m1} + g_{m2}$  and  $r_o \approx r_{o1} + r_{o2}$   
 (C)  $g_m \approx g_{m1}$  and  $r_o \approx r_{o1} \cdot g_{m2} \cdot r_{o2}$       (D)  $g_m \approx g_{m1}$  and  $r_o \approx r_{o2}$

[Ans. C\*]



$$g_m = \frac{\Delta I_D}{\Delta V_{in}} = \frac{I_D}{V_{gs}} = \frac{i_{D1}}{V_{gs}} = g_{m1}$$

To calculate  $r_o$ :



$$V_{\pi 2} = -I_x r_{o1}$$

$$I_x = g_{m2} V_{\pi 2} + \frac{(V_x - I_x r_{o1})}{r_{o2}}$$

$$I_x = -g_{m2} r_{o1} I_x + \frac{V_x}{r_{o2}} - I_x r_{o1} / r_{o2}$$

$$V_x = r_{o2} \left[ 1 + r_{o1} g_{m2} + \frac{r_{o1}}{r_{o2}} \right] I_x$$

$$r_o = \frac{V_x}{I_x} = r_{o1} + r_{o2} + r_{o1} r_{o2} g_{m2} \approx r_{o1} r_{o2} g_{m2}$$



3. Consider the following amplitude modulated signal:  
 $s(t) = \cos(2000\pi t) + 4 \cos(2400\pi t) + \cos(2800 \pi t)$   
 The ratio (accurate to three decimal places) of the power of the message signal to the power of the carrier signal is \_\_\_\_\_.

**[Ans. 0.25] Range: 0.12 to 0.13\***

$$s(t) = \cos(2000\pi t) + 4 \cos(2400\pi t) + \cos(2800 \pi t)$$

It can be compared with the standard form of the Am signal

$$s(t) = \frac{\mu A_c}{2} \cos[2\pi(f_c - f_m)t] + A_c \cos(2\pi f_c t) + \frac{\mu A_c}{2} \cos[2\pi(f_c + f_m)t]$$

By comparison, we get,  $A_c = 4$

$$\frac{\mu A_o}{2} = 1$$

$$\mu = \frac{2}{A_c}$$

$$\frac{A_m}{A_c} = \frac{2}{A_c} \Rightarrow A_m = 2$$

$$\frac{P_m}{P_c} = \frac{\frac{1}{2} A_m^2}{A_c^2} = \frac{A_m^2}{A_c^2} = \frac{(2)^2}{(4)^2} = \frac{1}{4} = 0.25$$

4. Let the input be  $u$  and the output be  $y$  of a system, and the other parameters are real constants. Identify which among the following systems is not a linear system:

(A)  $\frac{d^3y}{dt^3} + a_1 \frac{d^2y}{dt^2} + a_2 \frac{dy}{dt} + a_1 y = b_3 u + b_2 \frac{du}{dt} + b_1 \frac{d^2u}{dt^2}$  (With initial rest conditions)

(B)  $y(t) = \int_0^t e^{a(t-\tau)} \beta u(\tau) d\tau$

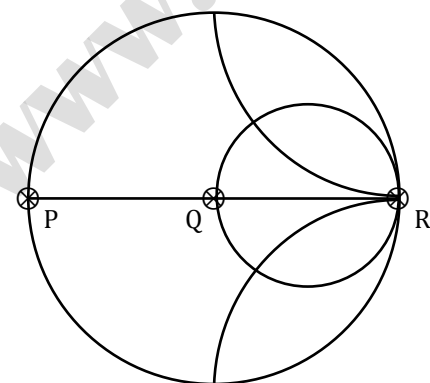
(C)  $y = au + b, b \neq 0$

(D)  $y = au$

**[Ans. C\*]**

$y = au + b, b \neq 0$  is a non-linear System

5. The points P, Q and R shown on the Smith chart (normalized impedance chart) in the following figure represent:



- (A) P: Open Circuit, Q: Short Circuit, R: Matched Load  
 (B) P: Open Circuit, Q: Matched Load, R: Short Circuit

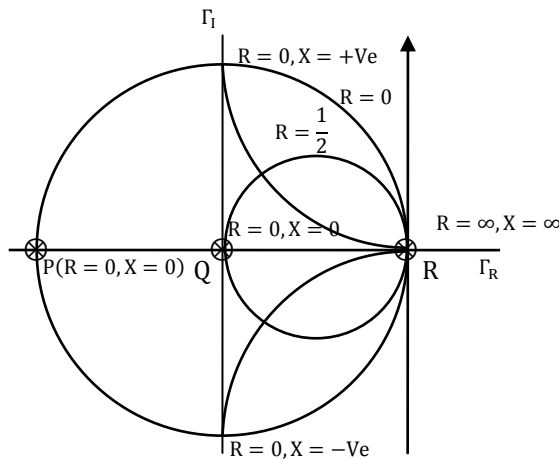
- (C) P: Short Circuit Q: Matched Load, R: Open Circuit  
(D) P: Short Circuit, Q: Open Circuit, R: Matched Load

**[Ans. C]**

Smith chart is used to find reflection coefficient and VSWR of a Transmission line.

$\frac{Z_L}{Z_0} \rightarrow$  Normalized impedance

$$\frac{Z_L}{Z_0} = R + jk$$



- At P:  $Z_L = 0 \Rightarrow$  Short circuit  
At Q:  $Z_L = Z_0 \Rightarrow$  Matched Load  
At R:  $Z_L = \infty \Rightarrow$  Open Circuit

6. Consider matrix  $A = \begin{bmatrix} k & 2k \\ k^2 - k & k^2 \end{bmatrix}$  and vector  $x = \begin{bmatrix} x_1 \\ x_2 \end{bmatrix}$ . The number of distinct real values of  $k$  for which the equation  $Ax = 0$  infinitely many solutions is \_\_\_\_\_.

**[Ans. \*] Range: 2 to 2**

$$A = \begin{bmatrix} k & 2k \\ k^2 - k & k^2 \end{bmatrix} \quad x = \begin{bmatrix} x_1 \\ x_2 \end{bmatrix}$$

$$Ax = 0$$

As 'x' has infinitely many solutions

$$\rho(A) < 2 \Rightarrow |A| = 0$$

$$k^3 - 2k(k^2 - k) = 0$$

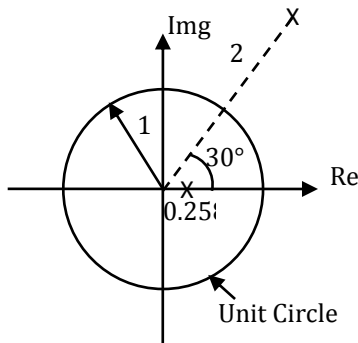
$$k^2(k - 2(k - 1)) = 0$$

$$k = 0, 0, 2$$

$\therefore$  Distinct values of  $k = 2$

7. A discrete-time all-pass system has two of its poles at  $0.25 \angle 0^\circ$  and  $2 \angle 30^\circ$ . Which one of the following statements about the system is TRUE?  
(A) It has two more poles at  $0.5 \angle 30^\circ$  and  $4 \angle 0^\circ$ .  
(B) It is stable only when the impulse response is two-sided.  
(C) It has constant phase response over all frequencies.  
(D) It has constant phase response over the entire z-plane.

**[Ans. B\*]**



The ROC should include unit circle to make the system stable. From the given pole pattern it is clear that, to make the system stable, the ROC should be finite circular strip or finite annular strip and hence the impulse response of the system should be also two-sided.

8. Taylor series expansion of  $f(x) = \int_0^x e^{-\frac{t^2}{2}} dt$  around  $x = 0$  has the form  $f(x) = a_0 + a_1x + a_2x^2 + \dots$ . The coefficient  $a_2$  (correct to two decimal places) is equal to \_\_\_\_\_.

[Ans. \*] Range: -0.01 to 0.01

$$f(x) = a_0 + a_1x + a_2x^2 + \dots \text{ Taylor series rep ... (1)}$$

$$f(x) = \int_0^x e^{-t^2/2} dt$$

Let  $g(t) = e^{-\frac{t^2}{2}}$  expansion around to  $g(0) = 1$

$$g'(t) = e^{-\frac{t^2}{2}}(-t) \Rightarrow g'(0) = 0$$

$$g''(t) = (-1)e^{-t/2} + e^{-\frac{t^2}{2}}(-t_1(-t))$$

$$g''(0) = -1$$

$$g(t) = 1 + \frac{t^2}{2!}(-1) + \dots$$

$$f(x) = \int_0^x \left( 1 + \frac{t^2}{2!} + \dots \right) dt = \left[ t + \frac{t^3}{6} + \dots \right]_0^x = x + \frac{x^3}{6} + \dots$$

$$f(x) = x + \frac{x^3}{6} + \dots \text{ (2)}$$

Comparing (1) and (2)

$$a_2 = 0$$

**Method 2:**

If  $f$  is continuous on  $(a, b)$  then

$$g(x) = \int_0^x f(t) dt \quad a \leq x \leq b$$

then  $g'(x) = f(x)$

Where  $g'(x)$  is continuous on  $(a, b)$  and differentiable on  $(a, b)$

$$f(x) = \int_0^x e^{-t^2/2} dt$$

$$f'(x) = e^{-x^2/2}$$

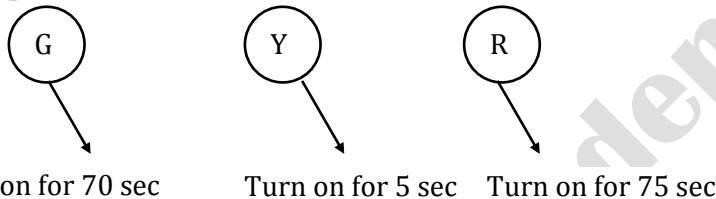
$$f''(x) = (-x)e^{-x^2/2}$$

$$a_2 = \frac{f''(0)}{2!} = 0$$

Coefficient of  $x^2$  term in Taylor series

9. A traffic signal cycles from GREEN to YELLOW, YELLOW to RED and RED to GREEN. In each cycle. GREEN is turned on for 70 seconds. YELLOW is turned on for 5 seconds and the RED is turned on for 75 seconds. This traffic light has to be implemented using a finite state machine (FSM). The only input to this. FSM is a clock of 5 second period. The minimum number of flip-flops required TO implement this FSM is \_\_\_\_\_.

[Ans. \*] Range: 5 to 5



Turn on for 70 sec      Turn on for 5 sec      Turn on for 75 sec

Given a clock of 5 second period

To get 70 seconds  $\Rightarrow$  counted number of numbers 't' be

$$\text{To get 58 seconds} \Rightarrow \frac{5 \text{ sec}}{5 \text{ sec}} = 1 \text{ number}$$

$$\text{To get 75 seconds} \Rightarrow \frac{75 \text{ sec}}{5 \text{ sec}} = 15 \text{ number}$$

$$\text{Total} = 14 + 1 + 15 = 30 \text{ numbers}$$

We should design a counter with 30 states

Number of flip-flops=5

$$2 \text{ flipflop} \rightarrow 2^2 \Rightarrow 4 \text{ states}$$

$$3 \text{ flipflop} \rightarrow 2^3 \Rightarrow 8 \text{ states}$$

$$5 \text{ flipflop} \rightarrow 2^5 \Rightarrow 32 \text{ states}$$

$$n=5$$

10. Let  $x(t)$  be a periodic function with period  $T = 10$ . The Fourier series coefficients for this series are denoted by  $a_k$  that is

$$X(t) = \sum_{k=-\infty}^{\infty} a_k e^{jk\frac{2\pi}{T}t}$$

The same function  $x(t)$  can also be considered as a periodic function with period  $T' = 40$ .

Let  $b_k$  be the Fourier series coefficients when period is taken as  $T'$ . IF  $\sum_{k=-\infty}^{\infty} |a_k| = 16$ ,

the  $\sum_{k=-\infty}^{\infty} b_k$  is equal to

(A) 256

(B) 64

(C) 16

(D) 4

[Ans. C\*]

Change in only time period or frequency does not change in the value of Fourier series coefficients

$$\text{So, } b_k = a_k$$

$$\sum_{k=-\infty}^{\infty} |b_k| = \sum_{k=-\infty}^{\infty} |a_k| = 16$$

11. Consider a binary channel code in which each code word has a fixed length of 5 bits. The Hamming distance between any pair of distinct code words in this code is at least 2. The maximum number of code words such a code can contain is\_\_\_\_\_.

**[Ans. \*] Range: 16 to 16**

Using singleton bound

$$k \leq n - d_{\min} + 1$$

Where

$d_{\min}$  = Minimum hamming distance between any pair of distinct codewords

$n$  = length of original codeword

$k$  = length of newly formed code word

$$k \leq 5 - 2 + 1$$

$$k \leq 4$$

Therefore, the maximum number of code words is  $2^4 = 16$

12. A good trans impedance amplifier has  
 (A) low input impedance and high output impedance,  
 (B) high input impedance and high output impedance.  
 (C) high input impedance and low output impedance.  
 (D) low input impedance and low output impedance.

**[Ans. D]**

A good trans-impedance amplifier should have low input impedance and low output impedance.

13. Let  $M$  be a real  $4 \times 4$  matrix. Consider the following statements:

S1:  $M$  has 4 linearly independent eigenvectors.

S2:  $M$  has 4 distinct eigenvalues.

S3:  $M$  is non-singular (invertible).

Which one among the following is TRUE?

(A) S1 implies S2

(B) S1 implies S3

(C) S2 implies S1

(D) S3 implies S2

**[Ans. C]**

For  $M$   $4 \times 4$  Matrix, if  $M$  has 4 linearly independent eigen vectors, then  $M$  has 4 distinct eigen vectors.

$\therefore$  Option (C) is correct

14. The Nyquist stability criterion and the Routh criterion both are powerful analysis tool for determining the stability of feedback controllers. Identify which of the following statements is FALSE:
- (A) Both the criteria provide information relative to the stable gain range of the system.
  - (B) The general shape of the Nyquist plot is readily obtained from the Bode magnitude plot for all minimum-phase systems.
  - (C) The Routh criterion is not applicable in the condition of transport lag which can be readily handled by the Nyquist criterion.
  - (D) The closed-loop frequency response for a unity feedback system cannot be obtained from the Nyquist plot.

[Ans. D]

15. A p-n step junction diode with a contact potential of 0.65 V has a depletion width of 1  $\mu\text{m}$  at equilibrium. The forward voltage (in volts, cancel to two decimal places) at which this width reduces to 0.6  $\mu\text{m}$  is \_\_\_\_\_.

[Ans. \*]Range: 0.40 to 0.43

No bias:  $V_0 = 0.65$   $W_1 = 1\mu\text{m}$

Forward bias  $V = ?$   $W_2 = 0.6\mu\text{m}$

$$W \propto \sqrt{(V_0 - V_f)}$$

$$\frac{W_2}{W_1} = \sqrt{\frac{V_0 - V_{f_2}}{V_0 - V_{f_1}}} = \sqrt{\frac{0.65 - V}{0.65}}$$

$$\frac{0.6}{1} = \sqrt{\frac{0.65 - V}{0.65}}$$

$$0.36 = \frac{0.65 - V}{0.65}$$

$$V = 0.65(1 - 0.36)$$

$$V = 0.416 \text{ V}$$

16. Let  $f(x, y) = \frac{ax^2 + by^2}{xy}$ , where a and b are constants. If  $\frac{\partial f}{\partial x} = \frac{\partial f}{\partial y}$  at  $x = 1$  and  $y = 2$ , then the relation between a and b s

(A)  $a = \frac{b}{4}$

(B)  $a = \frac{b}{2}$

(C)  $a = 2b$

(D)  $a = 4b$

[Ans. D]

$$f(x, y) = \frac{ax^2 + by^2}{xy}$$

$$\frac{\partial f}{\partial x} = \frac{1}{y} \left[ \frac{2ax(x) - (ax^2 + by^2)}{x^2} \right]$$

$$\left. \frac{\partial f}{\partial x} \right|_{(x=1, y=2)} = \frac{1}{2} \left[ \frac{2a(1)(1) - (a + 4b)}{1^2} \right] = \frac{a - 4b}{2}$$

$$\frac{\partial f}{\partial y} / (x = 1, y = 2) = \frac{1}{x} \left[ \frac{2by(y) - (ax^2 + by^2)}{y^2} \right]$$

$$= \frac{1}{1} \left[ \frac{2b(2)(2) - (a(1)^2 + b(4))}{2^2} \right] = \frac{4b - a}{4}$$

given  $\frac{\partial f}{\partial x} = \frac{\partial f}{\partial y} / (x = 1, y = 2)$

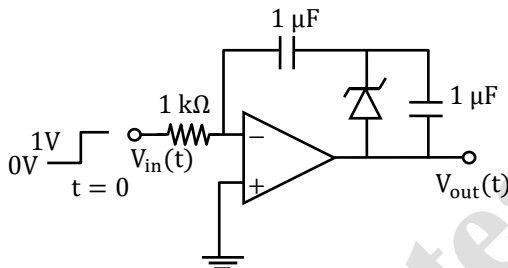
$$\frac{a - 4b}{2} = \frac{4b - a}{4}$$

$$2a - 8b = 4b - a$$

$$3a = 12b$$

$$a = 4b$$

17. In the circuit shown below, the op-amp is ideal and Zener voltage of the diode is 2.5 volt-At the input, unit step voltage is applied, i.e.  $V_{IN}(t) = u(t)$  volts. Also, at  $t = 0$ , the voltage across each of the capacitors is zero.

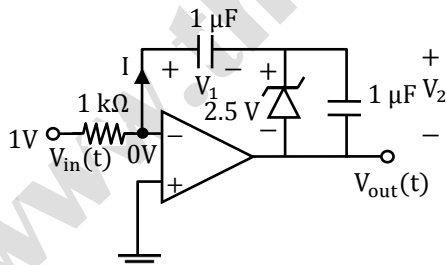


The time  $t$ , in milliseconds, at which the output voltage  $V_{out}$  crosses  $-10V$  is

- (A) 2.5 (B) 5  
(C) 7.5 (D) 10

[Ans. C\*]

For  $t > 0$



$$I = \frac{1V}{1k\Omega} = 1 \text{ mA}$$

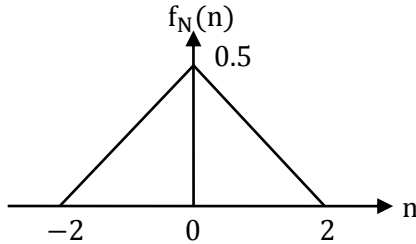
Till  $t = 2.5$  msec, both  $V_1$  and  $V_2$  will increase and after  $t = 2.5$  msec,  $V_2 = 2.5$  V and  $V_1$  increases with time.

When  $V_{out}(t) = -10$  V,  $V_1 = 7.5$  V

$$\text{So, } \frac{1}{1\mu\text{F}} \int_0^t (1\text{mA}) dt = 7.5 \text{ V}$$

$$10^3 t = 7.5; \quad t = 7.5 \text{ msec}$$

18. A binary source generates symbols  $X \in \{-1, 1\}$  which are transmitted over a noisy channel. The probability of transmitting  $X = 1$  is 0.5. Input to the threshold detector is  $R = X + N$ . The probability density function  $f_N(n)$  of the noise  $N$  is shown below,



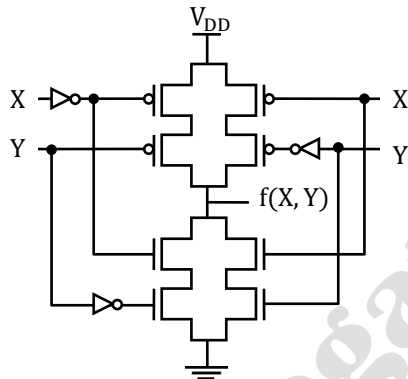
If the detection threshold is zero, then the probability of error (correct to two decimal places) is \_\_\_\_\_.

[Ans. \*] Range: 0.12 to 0.14

$$P_e = P\left(R > \frac{0}{X} = -1\right)P(X = -1) + P\left(R < \frac{0}{X} = 1\right)P(X = 1)$$

$$P_e = \frac{1}{2}(P(N > 1) + P(N < -1)); P_e = \frac{2}{2}P(N > 1) = 0.125$$

19. The logic function  $f(X, Y)$  realized by the given circuit is



- (A) NOR (B) AND  
(C) NAND (D) XOR

[Ans. D\*]

From pull-down network,

$$\overline{f(X, Y)} = \overline{X} \overline{Y} + XY = X \odot Y$$

$$f(X, Y) = \overline{X \odot Y} = X \oplus Y$$

20. There are two photolithography systems: one with light source of wavelength  $\lambda_1 = 156$  nm (System 1) and another with light source of wavelength  $\lambda_2 = 325$  nm (System 2). Both photolithography systems are otherwise identical. If the minimum feature sizes that can be realized using System 1 and System 2 are  $L_{\min 1}$  and  $L_{\min 2}$  respectively, the ratio  $L_{\min 1}/L_{\min 2}$  (correct to two decimal places) is \_\_\_\_\_.

[Ans. \*] Range: 0.47 to 0.51

In the concept of photolithographic systems, minimum feature size  $L_{\min} \propto$  wave length  $\lambda$

$$L_{\min} \propto \lambda$$

$$\frac{L_{\min 1}}{L_{\min 2}} = \frac{\lambda_1}{\lambda_2} = \frac{156}{325} = 0.48$$



21. A function  $F(A, B, C)$  defined by three Boolean variables A, B and C when expressed as sum of products is given by  $F = \bar{A} \cdot \bar{B} \cdot \bar{C} + \bar{A} \cdot B \cdot \bar{C} + A \cdot B \cdot \bar{C}$  where  $\bar{A}$ ,  $\bar{B}$ , and  $\bar{C}$  are the complements of the respective variables. The product of sums (POS) form of the function F is\_\_\_\_\_.

- (A)  $F = (A + B + C) \cdot (A + \bar{B} + C) \cdot (\bar{A} + B + C)$   
 (B)  $F = (\bar{A} + \bar{B} + \bar{C}) \cdot (\bar{A} + B + \bar{C}) \cdot (A + \bar{B} + \bar{C})$   
 (C)  $F = (A + B + C) \cdot (A + \bar{B} + \bar{C}) \cdot (\bar{A} + B + \bar{C}) \cdot (\bar{A} + \bar{B} + C) \cdot (\bar{A} + \bar{B} + \bar{C})$   
 (D)  $F = (\bar{A} + \bar{B} + C) \cdot (\bar{A} + B + C) \cdot (A + \bar{B} + C) \cdot (A + B + \bar{C}) \cdot (A + B + C)$

**[Ans. C]**

$$F(A, B, C) = \sum m(0, 2, 4)$$

$$F(A, B, C) = \pi M(1, 3, 5, 6, 7)$$

$$= (A + B + \bar{C})(A + \bar{B} + \bar{C})(\bar{A} + B + \bar{C})(\bar{A} + \bar{B} + C)(\bar{A} + \bar{B} + \bar{C})$$

22. A lossy transmission line has resistance per unit length  $R = 0.05 \Omega/\text{m}$ . The line is distortion less and has characteristic impedance of  $50 \Omega$ . The attenuation constant (in  $\text{Np}/\text{m}$ , correct to three decimal places) of the line is\_\_\_\_\_.

**[Ans. \*] Range: 0.001 to 0.001**

$$\text{Resistance } R = 0.05 \Omega/\text{m}$$

$$Z_0 = 50 \Omega$$

$$\alpha = \frac{R}{Z_0} = 0.001 \text{ Np}/\text{m}$$

$$\text{As } \alpha = \sqrt{RG} \quad Z_0 = \sqrt{\frac{R}{G}}$$

23. In a p-n junction diode at equilibrium, which one of the following statements is NOT TRUE?  
 (A) The hole and electron diffusion current components are in the same direction.  
 (B) The hole and electron drift current components are in the same direction,  
 (C) On an average, holes and electrons drift in opposite direction.  
 (D) On an average, electrons drift and diffuse in the same direction.

**[Ans. D]**

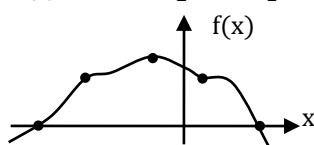
Option (D) is incorrect as drift of electrons is based on applied electric field whereas diffusion of electrons depend on concentration gradient.

24. Consider  $p(s) = s^3 + a_2s^2 + a_1s + a_0$  with all real coefficients. It is known that its derivative  $p'(s)$  has no real roots. The number of real roots of  $p(s)$  is

- (A) 0 (B) 1  
 (C) 2 (D) 3

**[Ans. B]**

$$P(s) = s^3 + a_2s^2 + a_1s + a_0$$



If two or more real roots exist for  $f(s)$ , then  $p'(s)$  will at least have '1' real root ( $P'(s) = 0$ )

∴ Real roots of P(s) is  $\leq 1$

Number of real roots = 1, As in cubic polynomial, there exists at least one real root

Option (B) is correct.

25. Let  $X_1, X_2, X_3$  and  $X_4$  be independent normal random variables with zero mean and unit variance. The probability that  $X_4$  is the smallest among the four is\_\_\_\_\_.

[Ans. \*] 0.25 to 0.25

$$P(X_4 \text{ is smallest}) = \frac{3!}{4!} = \frac{1}{4} = 0.25$$

**Q.26 - Q.55 Carry Two Mark each.**

26. The cutoff frequency of  $TE_{01}$  mode of an air filled rectangular waveguide having inner dimensions  $a$  cm X  $b$  cm ( $a > b$ ) is twice that of the dominant  $TE_{10}$  mode. When the waveguide is operated at a frequency which is 25% higher than the cutoff frequency of the dominant mode, the guide wavelength is found to be 4 cm. The value of  $b$  (in cm. correct to two decimal places) is\_\_\_\_\_.

[Ans. \*] Range: 0.7 to 0.8

Given  $f_{c(0,1)} = 2f_{c(1,0)}$

$$f_{c(m,n)} = \frac{c}{2} \sqrt{\left(\frac{m}{a}\right)^2 + \left(\frac{n}{b}\right)^2}$$

$$f_{c(0,1)} = \frac{c}{2b}$$

$$\frac{c}{2b} = 2 \left[ \frac{c}{2a} \right] \Rightarrow a = 2b$$

As  $a > b$   $TE_{10}$  is dominant mode

Given  $\lambda_g = 4$  cm

$$\lambda_g = \frac{\lambda}{\cos \theta} = \frac{\lambda}{\sqrt{1 - \left(\frac{f_c}{f}\right)^2}}$$

Given  $f = \frac{5}{4} f_c$

$$\Rightarrow \frac{f_c}{f} = \frac{4}{5}$$

$$\lambda_g = \frac{\lambda}{\sqrt{1 - \left(\frac{4}{5}\right)^2}} = \frac{\lambda}{\left(\frac{3}{5}\right)}$$

$$\frac{4 \times 3}{5} = \lambda = \frac{c}{f}$$

$$\lambda = \frac{c}{f} = \frac{c}{\frac{5}{4} f_c} = \frac{c}{\frac{5}{4} \times \frac{c}{2b}} = \frac{8a}{5}$$

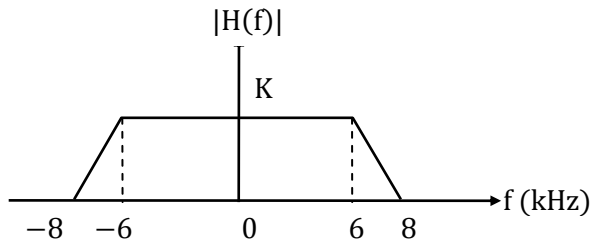
$$\frac{12}{5} = \frac{8a}{5}$$

$$\Rightarrow a = \frac{3}{2}$$

$$\Rightarrow b = \frac{a}{2} = \frac{3}{4}$$

$$b = 0.75 \text{ cm}$$

27. A band limited low-pass signal  $x(t)$  of bandwidth 5 kHz is sampled at a sampling rate  $f_s$ . The signal  $x(t)$  is reconstructed using the reconstruction filter  $H(f)$  whose magnitude response is shown below:



The minimum sampling rate  $f_s$  (in kHz) for perfect reconstruction of  $x(t)$  is \_\_\_\_\_.

[Ans. \*] Range: 13 to 13

Continuous time signal  $x(t)$  band limited to 5 kHz

To reconstruct a Continuous times signal sampling frequency  $f_s$

$$f_c \leq f_s$$

$$f_c \leq f_s - f_m$$

$$f_c = 8 \text{ kHz}$$

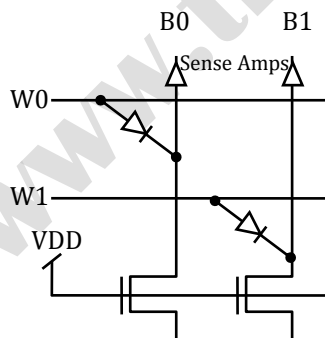
$$f_m = 5 \text{ kHz}$$

$$f_c = f_s - 5$$

$$8 = f_s - 5$$

$$f_s = 13 \text{ kHz}$$

28. A  $2 \times 2$  ROM array is built with the help of diodes as shown in the circuit below. Here  $W_0$  and  $W_1$  are signals that select the word lines and  $B_0$  and  $B_1$  are signals that are output of the sense amps based on the stored data corresponding to the bit lines during the read operation.



$$\begin{matrix} B_0 & B_1 \\ W_0 [D_{00} & D_{01}] \\ W_1 [D_{10} & D_{11}] \end{matrix}$$

Bits stored in the Rom Array

During the read operation, the selected word line goes high and the other word line is in a high impedance state. As per the implementation shown in the circuit diagram above, what are the bits corresponding to  $D_{ij}$  (where  $i = 0$  or  $1$  and  $j = 0$  or  $1$ ) stored in the ROM?

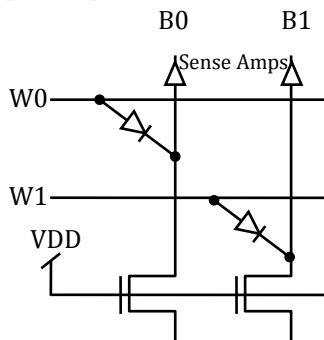
(A)  $\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$

(B)  $\begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix}$

(C)  $\begin{bmatrix} 1 & 0 \\ 1 & 0 \end{bmatrix}$

(D)  $\begin{bmatrix} 1 & 1 \\ 0 & 0 \end{bmatrix}$

[Ans. A]



• When,  $W_0 = V_{DD}$ ,  $B_0 = V_{DD}$ ; otherwise  $B_0 = 0$

• When  $W_1 = V_{DD}$ ,  $B_1 = V_{DD}$ ; otherwise  $B_1 = 0$

So,  $B_0 = W_0$  and  $B_1 = W_1$

Hence,  $\begin{matrix} B_0 & B_1 \\ W_0 & \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} \\ W_1 & \end{matrix}$

29. Let  $r = x^2 + y - z$  and  $z^3 - xy + yz + y^3 = 1$ . Assume that  $x$  and  $y$  are independent variables. At  $(x, y, z) = (2, -1, 1)$  the value (correct to two decimal places) of  $\frac{\partial r}{\partial x}$  is

[Ans. \*] Range: 4.4 to 4.6

Given  $r = x^2 + y - z \dots$  (1)

$z^3 - xy + yz + y^3 = 1 \dots$  (2)

Apply partial derivative on (2) wrt  $x$

$$3z^2 \frac{\partial z}{\partial x} - y + y \frac{\partial z}{\partial x} = 0$$

$$\frac{\partial z}{\partial x} = \frac{y}{3z^2 + y}$$

$$\left. \frac{\partial z}{\partial x} \right|_{P(2,-1,1)} = \frac{-1}{3(1) - 1} = \frac{-1}{2}$$

Apply partial derivative wrt on (1)

$$\frac{\partial r}{\partial x} = 2x - \frac{\partial z}{\partial x}$$

$$\left. \frac{\partial r}{\partial x} \right|_{(2,-1,1)} = 2(2) - \left( \frac{-1}{2} \right) = 4.5$$

30. Red (R), Green (G) and Blue (B) Light Emitting Diodes (LEDs) were fabricated using p-n junctions of three different inorganic semiconductors having different band-gaps. The

built-in voltages of red, green and blue diodes are  $V_R, V_G$  and  $V_B$ , respectively. Assume donor and acceptor doping to be the same ( $N_A$  and  $N_D$ , respectively) in the p and n sides of all the three diodes. Which one of the following relationships about the built-in voltages is TRUE?

- (A)  $V_R > V_G > V_B$  (B)  $V_R < V_G < V_B$   
(C)  $V_R = V_G = V_B$  (D)  $V_R > V_G < V_B$

**[Ans. B\*]**

$$\lambda_R > \lambda_G > \lambda_B$$

$$\text{Energy gap, } E_g \propto \frac{1}{\lambda}$$

$$\text{So, } E_{gR} < E_{gG} < E_{gB}$$

Materials with high energy gap will have built in voltages, when doping concentrations are same.

$$\text{So, } V_R < V_G < V_B$$

31. Let  $c(t) = A_c \cos(2\pi f_c t)$  and  $m(t) = \cos(2\pi f_m t)$ . It is given that  $f_c \gg 5f_m$ . The signal  $c(t) + m(t)$  is applied to the input of a non-linear device, whose output  $V_o(t)$  is related to the input  $V_i(t)$  as  $V_o(t) = aV_i(t) + bV_i^2(t)$ , where  $a$  and  $b$  are positive constants. The output of the non-linear device is passed through an ideal band-pass filter with center frequency  $f_c$  and bandwidth  $3f_m$ , to produce an amplitude modulated (AM) wave. If it desired to have the sideband power of the AM wave to be half of the earlier power, then  $a/b$  is

- (A) 0.25 (B) 0.5  
(C) 1 (D) 2

**[Ans. D]**

$$v_o(t) = av_i(t) + bv_i^2(t)$$

$$= a(A_c \cos(2\pi f_c t) + A_m \cos(2\pi f_m t)) + b(A_c \cos(2\pi f_c t) + A_m \cos(2\pi f_m t))^2$$

The output of BPF is given by

$$y_{\text{BPF}}(t) = aA_c \left[ 1 + \frac{2b}{a} \cos(2\pi f_m t) \right] \cos(2\pi f_c t)$$

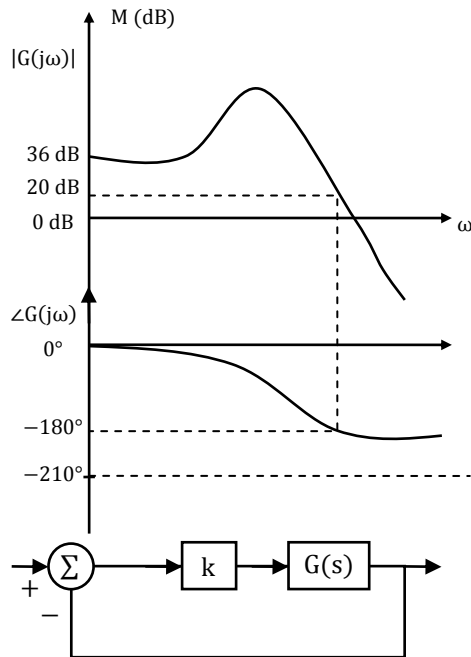
Given sideband power is half of the carrier power

$$P_{\text{SB}} = \frac{\mu^2 P_c}{2} = \frac{P_c}{2} = \mu = 1$$

$$\frac{2b}{a} = 1 \Rightarrow \frac{a}{b} = 2$$

32. The figure below shows the Bode magnitude and phase plots of a stable transfer function

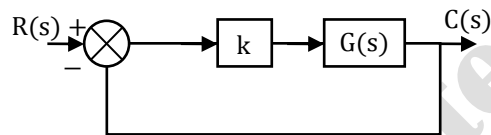
$$G(s) = \frac{n_0}{s^3 + d_1 s^2 + d_1 s + d_0}$$



Consider the negative unity feedback configuration with gain  $k$  in the feed forward path. The closed loop is stable for  $k < k_0$ . The maximum value of  $k_0$  is

**[Ans. \*] Range: 0.1 to 0.1**

$$G(s) = \frac{n_0}{s^3 + d_1 s^2 + d_1 s + d_0}$$



From diagram

$$|G(j\omega)H(j\omega)|_{dB} = 20 \text{ dB}$$

$$a + \omega = \omega_{pc}$$

$$|G(j\omega)| = k - G(j\omega)$$

To the closed loop system to be stable

Gain margin  $> 1$

$$\frac{1}{|G(j\omega)H(j\omega)|_{\omega=\omega_{pc}}} > 1$$

$$|G(j\omega)|_{\omega_{pc}} < 1 \text{ or } \log(1) = 0 \text{ dB}$$

$$20 \log k + 20 \log k < 0 \text{ dB}$$

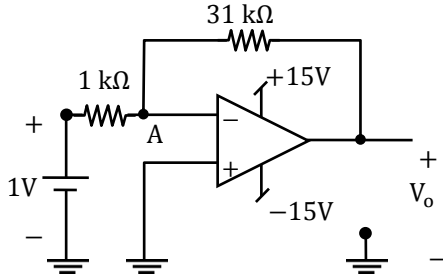
$$20 \log k < -20 \text{ dB}$$

$$\log k < -1$$

Hence maximum value of  $k$  is  $k_0$  for when is stable

$$k_{\max} = k_0 = 10^{-1} = 0.1$$

33. An op-amp based circuit is implemented as shown below.



In the above circuit, assume the op-amp to be ideal. The voltage (in volts, correct to one decimal place) at node A, connected to the negative input of the op-amp as indicated in the figure is\_\_\_\_\_.

[Ans. \*] Range: 0.4 to 0.6

Applying the concept of virtual ground, we get

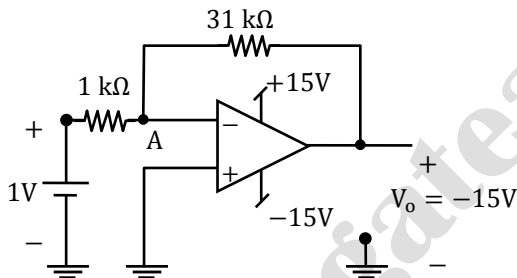
$$V_o = -\frac{R_2}{R_1} \cdot V_{in} \quad [\because \text{non-inverting amplifier}]$$

$$\therefore V_o = -\frac{31\text{k}\Omega}{1\text{k}\Omega} \times 1\text{V}$$

$$V_o = -31\text{V} < -15\text{V}$$

Which is not possible

Hence. The output voltage of the op-amp is equal to  $-15\text{V}$



Now applying KCL of node 'A', we get

$$\frac{V_A - (-15)}{31\text{k}\Omega} + \frac{V_A - 1}{1\text{k}\Omega} = 0$$

$$\frac{V_A}{31\text{k}\Omega} + \frac{V_A}{1\text{k}\Omega} = \frac{-15}{31\text{k}\Omega} + \frac{1}{1\text{k}\Omega}$$

$$V_A \left[ \frac{1}{31} + \frac{1}{1} \right] = \frac{-15}{31} + 1$$

$$V_A = 0.5\text{V}$$

34. For a unity feedback control system with the forward path transfer function

$$G(s) = \frac{K}{s(s+2)}$$

The peak resonant magnitude  $M_r$  of the closed-loop frequency response is 2.

The corresponding value of the gain K (correct to two decimal places) is

[Ans. \*] Range: 14 to 17

$$G(s) = \frac{k}{s(s+2)}$$

Standard  $G(s)$  of 2<sup>nd</sup> order system

$$G(s) = \frac{\omega_n^2}{s(s + 2\xi\omega_n)}$$

$$\omega_n = \sqrt{k} \dots \dots \textcircled{1}$$

$$2\xi\omega_n = 2$$

$$\xi = \frac{1}{\omega_n} \dots \dots \textcircled{2}$$

$$\text{Resonant peak } M_r = \frac{1}{2\xi\sqrt{1-\xi^2}}$$

$$M_r = 2$$

$$2 = \frac{1}{2\xi\sqrt{1-\xi^2}}$$

$$\xi^4 - \xi^2 + \frac{1}{4} = 0$$

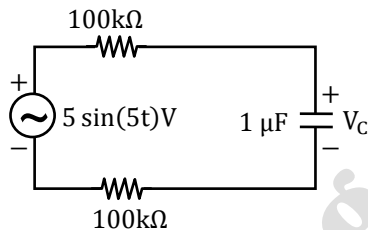
$$\xi^2 = 0.0667, 0.92$$

$$\xi = \sqrt{0.0667} = 0.258$$

$$\omega_n = \frac{1}{\xi} = 3.875$$

$$k = \omega_n^2 = (3.875)^2 = 15.02$$

35. For the circuit given in the figure, the voltage  $V_C$  (in volts) across the capacitor is



(A)  $1.25 \sqrt{2} \sin [5t - 0.25 \pi]$

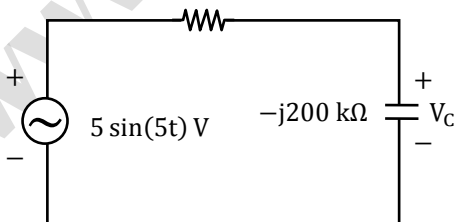
(B)  $1.25 \sqrt{2} \sin [5t - 0.125\pi]$

(C)  $2.5 \sqrt{2} \sin [5t - 0.25\pi]$

(D)  $2.5\sqrt{2} \sin [5t - 0.125 \pi]$

[Ans. C\*]

$$\frac{1}{\omega C} = \frac{1}{5 \times 10^{-6}} = 200 \text{ k}\Omega$$



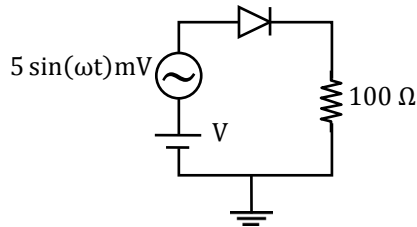
$$V_C = \frac{5 \angle 0^\circ}{200 - j200} \times (-j200) \text{ V} = \frac{5 \angle 0^\circ 1 \angle -90^\circ}{\sqrt{2} \angle -45^\circ} \text{ V}$$

$$= \frac{5}{\sqrt{2}} \angle -45^\circ \text{ V} = 2.5\sqrt{2} \sin \left( 5t - \frac{\pi}{4} \right) \text{ V}$$

$$= 2.5\sqrt{2} \sin (5t - 0.25\pi) \text{ V}$$



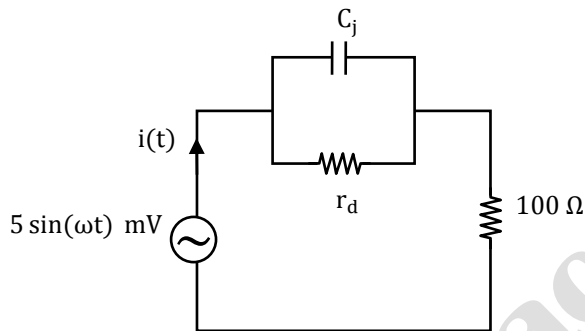
36. A dc current of  $26 \mu\text{A}$  flows through the circuit shown. The diode in the circuit is forward biased and it has an ideality factor of one. At the quiescent point, the diode has a junction capacitance of  $0.5 \text{ nF}$ , Its neutral region resistances can be neglected. Assume that the room temperature thermal equivalent voltage is  $26 \text{ mV}$ .



For  $\omega = 2 \times 10^6 \text{ rad/s}$ , the amplitude of the small-signal component of diode current (in  $\mu\text{A}$ , correct to one decimal place) is \_\_\_\_\_.

[Ans. \*] Range: 6.2 to 6.6\*

The small signal equivalent model of the given circuit can be drawn as shown below.



Given that,  $\omega = 2 \times 10^6 \text{ rad/sec}$

$$C_j = 0.5 \text{ nF}$$

$$I_{DC} = 26 \mu\text{A}$$

$$V_T = 26 \text{ mV}$$

$$\eta = 1$$

$$\text{So, } r_d = \frac{\eta V_T}{I_{DC}} = \frac{26 \text{ mV}}{26 \mu\text{A}} = 1 \text{ k}\Omega$$

$$\frac{1}{\omega C_j} = \frac{1}{2 \times 10^6 \times 0.5 \times 10^{-9}} \Omega = 1 \text{ k}\Omega$$

So, total impedance of the circuit will be

$$Z = \left( r_d \parallel \frac{1}{j\omega C_j} \right) + 100 \Omega$$

$$\left( r_d \parallel \frac{1}{j\omega C_j} \right) = \frac{(1000) - (-j1000)}{1000 - j1000} \Omega = \frac{-j(1+j)}{2} \text{ k}\Omega$$

$$= \frac{1}{2} (1-j) \text{ k}\Omega = (500 - j500) \Omega$$

$$\therefore Z = 600 - j500 \Omega$$

$$|Z| = 100\sqrt{36 + 25} = 100\sqrt{61} \Omega$$

$$I_m = \frac{V_m}{|Z|} = \frac{5 \text{ mV}}{100\sqrt{61} \Omega} = \frac{50}{\sqrt{61}} \mu\text{A} = 6.40 \mu\text{A}$$

37. Consider a white Gaussian noise process  $N(t)$  with two-sided power spectral density  $S_N(f) = 0.5 \text{ W/Hz}$  as input to a filter with impulse response  $0.5e^{-t^2/2}$  (where  $t$  is in seconds) resulting in output  $Y(t)$ . The power in  $Y(t)$  in watts is  
 (A) 0.11 (B) 0.22  
 (C) 0.33 (D) 0.44

**[Ans. B]**

Output power = Area under output power spectral density

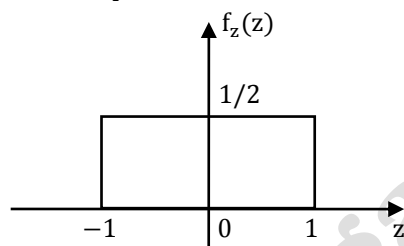
Output power spectral density =  $S_N(f)|H(f)|^2$

$$\text{Output power} = \int_{-\infty}^{\infty} e^{-f^2} df = \frac{\sqrt{\pi}}{8} = 0.22$$

38. A random variable  $X$  takes values  $-0.5$  and  $0.5$  with probabilities  $\frac{1}{4}$  and  $\frac{3}{4}$  respectively. The noisy observation of  $X$  is  $Y = X + Z$ , where  $Z$  has uniform probability density over the interval  $(-1,1)$ .  $X$  and  $Z$  are independent. If the MAP rule based detector outputs  $\hat{X}$  as  $\hat{X} = \begin{cases} -0.5 & Y < \alpha \\ 0.5 & Y \geq \alpha \end{cases}$  then the value of  $\alpha$  (accurate to two decimal places) is \_\_\_\_\_.

**[Ans. \*] Range: -0.5 to -0.5\***

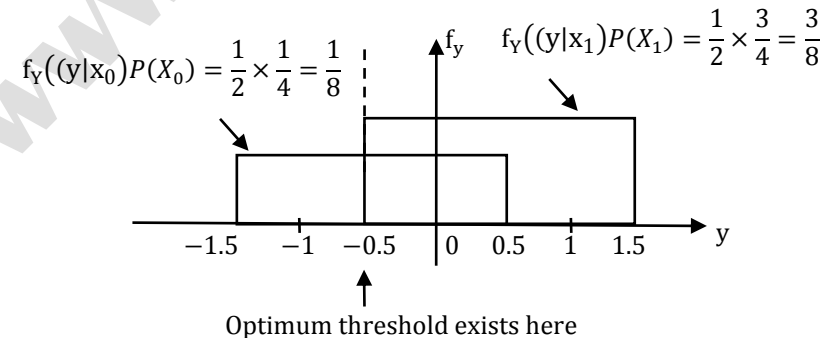
$$P_{(x_0)} = \frac{1}{4}$$



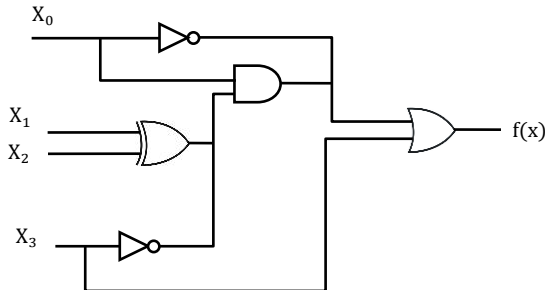
$$P_{(x_1)} = \frac{3}{4}$$

$$\text{MAP criteria, } f_Y(y|x_0)P(x_0) > f_Y(y|x_1)P(x_1)$$

So,  $\alpha = -0.50$



39. The logic gates shown in the digital circuit below use strong pull-down NMOS transistors for LOW logic level at the outputs. When the pull-downs are off, high-value resistors set the output logic levels to HIGH (i.e. the pull-ups are weak). Note That some nodes are intentionally shorted to implement “wired logic”. Such shorted nodes will be HIGH only if the outputs of all the gates whose outputs are shorted are HIGH.



The number of distinct values of  $X_3X_2X_1X_0$  (out of the 16 possible values) that give  $Y = 1$  is \_\_\_\_\_.

[Ans. \*] Range: 8 to 8

$X_0X_1X_2X_3$   
1

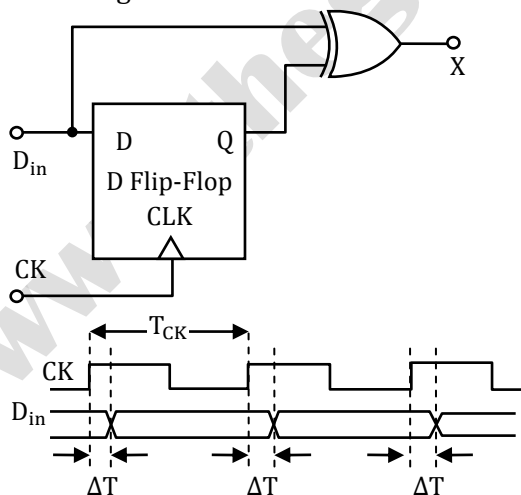
If  $X_3 = 0$ , then band on  $X_1$

if  $X_1 = 0$  output is 0

$X_1 = 1$  output is 0

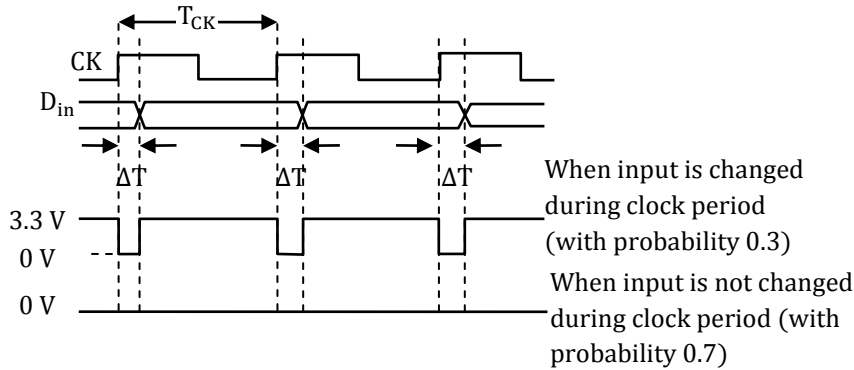
So only 8 inputs

40. In the circuit shown below, a positive edge-Triggered D Flip-Flop is used for sampling input data  $D_{in}$  using clock CK. The XOR gate outputs 3.3 volts for logic HIGH and 0 volts for logic LOW levels. The data bit and clock periods are equal and the value of  $\Delta/T_{CK} = 0.15$ . where the parameters  $\Delta T$  and  $T_{CK}$  are shown in the figure. Assume that the Flip-Flop and the XOR gate are ideal.



If the probability of input data bit ( $D_{in}$ ) transition in each clock period is 0.3. The average value (in volts, accurate to two decimal places) of the voltage at node X, is \_\_\_\_\_.

[Ans. \*] Range: 0.82 to 0.86\*

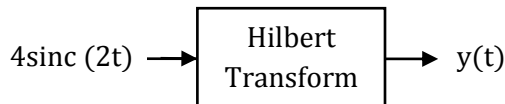


$$V_{x(\text{avg})} = \left[ 0.3 \times 3.3 \left( 1 - \frac{\Delta T}{T_{\text{CK}}} \right) \right] + [0.7 \times 0]V$$

$$= 0.3 \times 3.3 \times (1 - 0.15)V$$

$$= 0.3 \times 3.3 \times 0.85 V = 0.8415V$$

41. The input  $4 \text{ sinc}(2t)$  is fed to a Hilbert transformer to obtain  $y(t)$  as shown in the figure below:



Here  $\text{sinc}(x) = \frac{\sin(\pi x)}{\pi x}$ . The value (accurate to two decimal places) of  $\int_{-\infty}^{\infty} |y(t)|^2 dt$  is \_\_\_\_.

**[Ans. \*] Range: 8 to 8**

$$x(t) = 4 \text{ sinc}(2t)$$

$$x(t) = 4 \text{ Sa}(2\pi t)$$

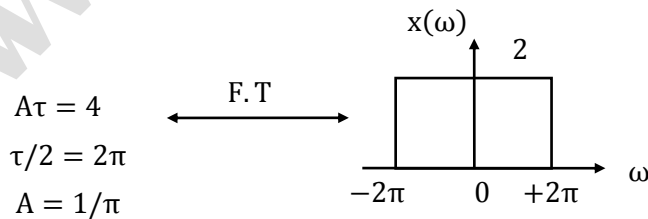
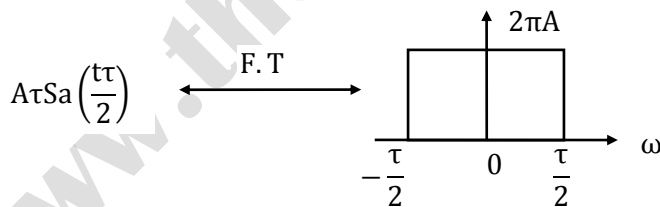
$$y(t) = \text{Hilbert transform of } x(t)$$

$$y(t) = \hat{x}(t)$$

$$\text{Energy of } x(t) = \text{Energy of Hilbert transform of } x(t)$$

$$x(t) \rightarrow E = \text{Energy of } x(t)$$

$$x(t) = 4 \text{ Sa}(2\pi t)$$



$$1 \int_{-2}^2 |y(t)|^2 dt \rightarrow \frac{1}{2\pi} \int_{-2\pi}^{2\pi} (2)^2 d\omega$$

$$\Rightarrow \frac{4}{2\pi} (2\pi - (-2\pi))$$

$$\Rightarrow \frac{4 \times 4\pi}{2\pi} = 8$$

42. The position of a particle  $y(t)$  is described by the differential equation:

$$\frac{d^2y}{dt^2} = -\frac{dy}{dt} - \frac{5y}{4}$$

The initial conditions are  $y(0) = 1$  and  $\left. \frac{dy}{dt} \right|_{t=0} = 0$ . The position (accurate to two decimal places) of the particle at  $t = \pi$  is \_\_\_\_\_.

**[Ans. \*] Range: -0.23 to -0.19**

$$\frac{d^2y}{dt^2} = -\frac{dy}{dt} - \frac{5y}{4} = 0$$

$$D = \frac{d}{dt}, D^2 = \frac{d^2}{dt^2}$$

$$(D^2 + D + \frac{5}{4})y = 0$$

$$C.S = C.F + P.I = 0$$

Complete solution is complementary function auxiliary equation

$$D^2 + D + \frac{5}{4} = 0$$

$$D = \frac{-1 \pm \sqrt{1^2 - 4 \times \frac{5}{4}}}{2}$$

$$= \frac{-1 \pm 2j}{2} = -\frac{1}{2} \pm j$$

$$C.F = e^{-\frac{1}{2}t} (C_1 \cos 1t + C_2 \sin 1t)$$

$$C.S = y = e^{-\frac{1}{2}t} (C_1 \cos t + C_2 \sin t)$$

$$\text{Given } y(0) = 1$$

$$1 = C_1$$

$$\Rightarrow C_1 = 1$$

$$\frac{dy}{dt} = e^{-\frac{1}{2}t} \left( -\frac{1}{2} \right) (\cos t + C_2 \sin t) + e^{-\frac{1}{2}t} (-\sin t + C_2 \cos t)$$

$$0 = -\frac{1}{2}(1) + (C_2)$$

$$C_2 = \frac{1}{2}$$

$$y = e^{-\frac{1}{2}t} \left( \cos t + \frac{1}{2} \sin t \right)$$

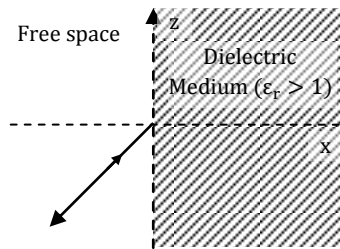
$$\frac{y}{\text{at } x = \pi} = e^{-\frac{\pi}{2}} (-1 + 0)$$

$$= -e^{-\frac{\pi}{2}} = -0.207$$

43. A uniform plane wave traveling in free space and having the electric field

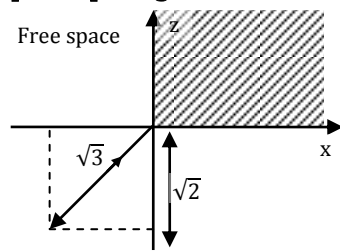
$$\vec{E} = (\sqrt{2}\hat{a}_x - \hat{a}_z) \cos[6\sqrt{3}\pi \times 10^8 t - 2\pi(x + \sqrt{2}z)] \text{ V/m}$$

is incident on a dielectric medium (relative permittivity  $> 1$ , relative permeability-1) as shown in the figure and there is no reflected wave.



The relative permittivity (correct to two decimal places) of the dielectric medium is \_\_\_\_\_.

**[Ans. \*] Range: 1.9 to 2.1**



$$\sin \phi = \frac{\sqrt{2}}{\sqrt{3}}$$

At zero reflection

$$\tan \phi = \sqrt{\frac{\epsilon_{r2}}{\epsilon_1}} \Rightarrow \phi = 54.73^\circ$$

$$\tan(54.73) = \sqrt{\frac{\epsilon_r}{\epsilon_{r1}}}$$

$$\sqrt{\frac{\epsilon_r}{(1)}} = \sqrt{2}$$

$$\Rightarrow \epsilon_r = 2$$

44. Let  $X[k] = k + 1, 0 \leq k \leq 7$  be 8-point DFT of a sequence  $x[n]$  where  $X[k] = \sum_{n=0}^{N-1} x[n] e^{-j2\pi nk/N}$

The value (Correct to two decimal places) of  $\sum_{n=0}^3 x[2n]$  is \_\_\_\_\_

**[Ans. \*] Range: 2.90 to 3.10**

$$\sum_{n=0}^3 x(2n) = x(0) + x(2) + x(4) + x(6)$$

$$x(k) = k + 1 \Rightarrow \text{for } 0 \leq k \leq 7$$

$$x(0) = 1$$

$$x(1) = 2$$

$$x(2) = 3$$

$$x(3) = 4$$

$$x(4) = 5$$

$$x(5) = 6$$

$$x(6) = 7$$

$$x(7) = 8$$

$$x(k) = \sum_{n=0}^7 x(n) e^{-\frac{j2\pi}{8}kn}; k = 0$$

$$x(0) = \sum_{n=0}^7 x(n) \Rightarrow x(0) + x(1) + x(2) + x(3) + x(4) + x(5) + x(6) + x(7) \dots \dots \textcircled{1}$$

We know that

$$x\left(\frac{N}{2}\right) = \sum_{n=0}^{N-1} x(n)(-1)^n$$

$$x(4) = \sum_{n=0}^7 x(n)(-1)^n$$

$$\Rightarrow x(0) - x(1) + x(2) - x(3) + x(4) - x(5) + x(6) - x(7) \dots \dots \textcircled{2}$$

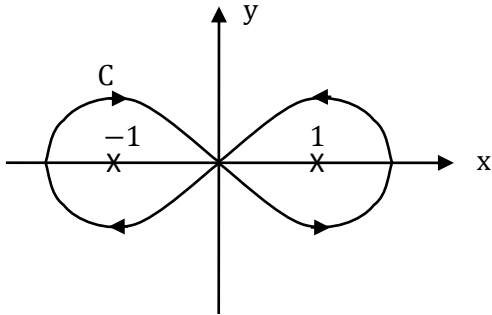
Sum of  $\textcircled{1}$  and  $\textcircled{2}$

$$x(0) + x(4) = 2((x(0) + x(2) + x(4) + x(6)))$$

$$\frac{x(0) + x(4)}{2} = \sum_{n=0}^3 x(en)$$

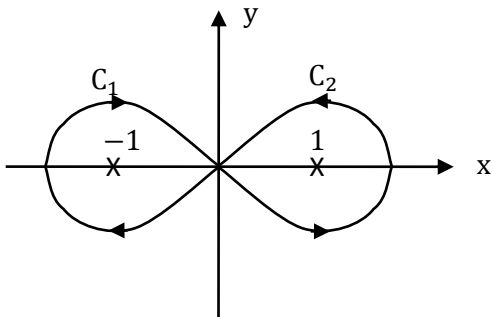
$$\frac{1 + 5}{2} = \sum_{n=0}^3 x(n) = 3$$

45. The contour C given below is on the complex plane  $z = x + jy$ , where  $\sqrt{-1} = j$



The value of the integral  $\frac{1}{\pi j} \oint_C \frac{dz}{z^2 - 1}$  is \_\_\_\_\_

[Ans. \*] Range: 2 to 2



$$\frac{1}{\pi j} \oint_C \frac{dz}{z^2 - 1} = ?$$

$$\frac{1}{\pi j} \oint_C \frac{dz}{(z-1)(z+1)}$$

$$\frac{1}{2\pi j} \left[ \oint_{C_2} \frac{dz}{z-1} - \oint_{C_1} \frac{dz}{z+1} \right]$$

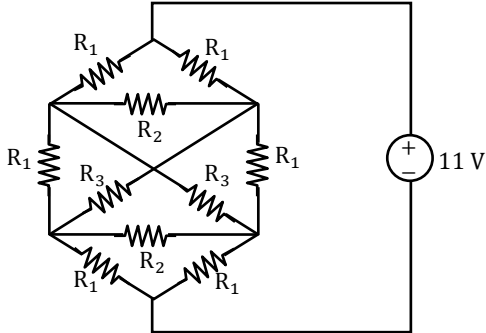
$$\frac{1}{2\pi j} \left[ \oint_{C_2} \frac{dz}{z-1} - (-1) \oint_{C_1} \frac{dz}{z+1} \right]$$

$$\frac{1}{2\pi j} [2\pi j + 2\pi j] = 2$$

$$\left[ \oint_{ACW} f(z) dz = - \oint_{CW} f(z) dz \right]$$

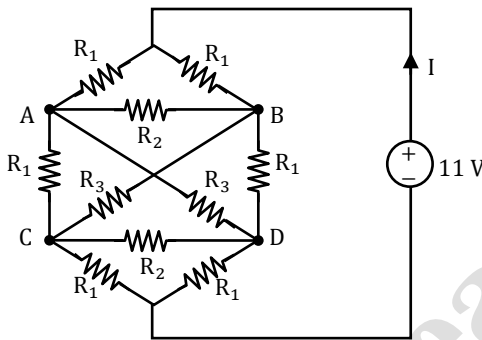


46. Consider the network shown below with  $R_1 = 1\Omega$ ,  $R_2 = 2\Omega$  and  $R_3 = 3\Omega$ . The network is connected to a constant voltage source of 11V.



The magnitude of the current (in amperes, accurate 10 two decimal places) through the source is \_\_\_\_\_.

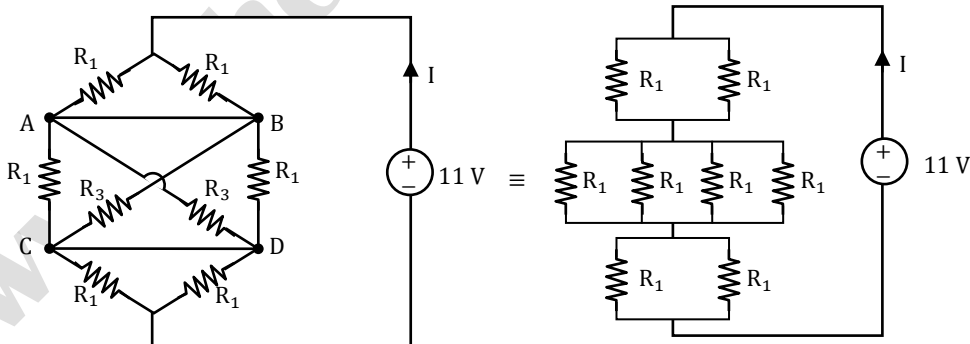
[Ans. \*]Range: 7.9 to 8.1\*



As the network is symmetric,

$$V_A = V_B \text{ and } V_C = V_D$$

So, current through  $R_2$  resistors is zero and as  $V_A = V_B$  and  $V_C = V_D$ , electrically the circuit can be reduced as,



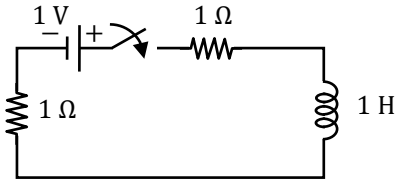
$$\text{Total resistances, } R_T = 2(R_1 \parallel R_1) + (R_1 \parallel R_1 \parallel R_3 \parallel R_3) = R_1 + \left( \frac{R_1}{2} \parallel \frac{R_3}{2} \right)$$

Given that  $R_1 = 1\Omega$  and  $R_3 = 3\Omega$

$$\text{So, } R_T = 1 + \left( \frac{1}{2} \parallel \frac{3}{2} \right) \Omega = 1 + \frac{3/2}{4} = \frac{11}{8} \Omega$$

$$I = \frac{11V}{R_T} = \frac{11}{(11/8)} = 8A$$

47. For the circuit given in the figure, the magnitude of the loop current (in amperes, correct to three decimal places) 0.5 second after closing the switch is\_\_\_\_\_.



[Ans. \*]Range: 0.284 to 0.348

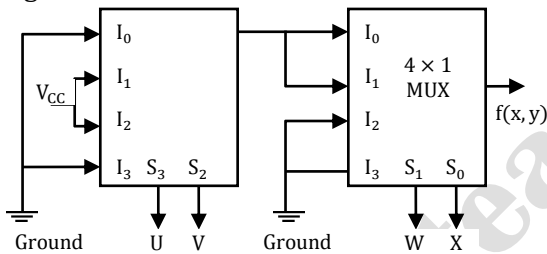
$$\text{Loop current, } i(t) = \frac{1}{1+1} (1 - e^{-t/\tau}) \text{A}; t > 0$$

$$\tau = \frac{L}{R_{eq}} = \frac{1}{1+1} = \frac{1}{2} \text{sec}$$

$$i(t) = \frac{1}{2} (1 - e^{-2t}) \text{A}; t > 0$$

$$\text{at } t = 0.5 \text{ sec, } i(t) = \frac{1}{2} (1 - e^{-1}) \text{A} = 0.316 \text{ A}$$

48. A four-variable Boolean function is realized using 4 x 1 multiplexers as shown in the figure.



The minimized expression for  $F(U, V, W, X)$  is

(A)  $(UV + \bar{U}\bar{V})\bar{W}$

(B)  $(UV + \bar{U}\bar{V})(\bar{W}\bar{X} + \bar{W}X)$

(C)  $(U\bar{V} + \bar{U}V)\bar{W}$

(D)  $(U\bar{V} + \bar{U}V)(\bar{W}\bar{X} + \bar{W}X)$

[Ans. C]

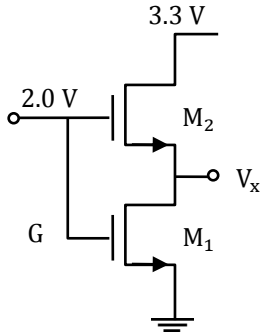
Let output of first MUX is  $z$

$$\text{Thus } F(u, v, w, x) = \bar{w}\bar{x}(z) + \bar{w}x(z) + 0 + 0$$

$$z = \bar{u}v(1) + u\bar{v}$$

$$= \bar{w}z(\bar{x} + x) = \bar{w}z = \bar{w}(\bar{u}v + u\bar{v})$$

49. In the circuit shown below, the  $(W/L)$  value for  $M_2$  is twice that for  $M_1$ . The two nMOS transistors are otherwise identical. The threshold voltage  $V_T$  for both transistors is 1.0V. Note that  $V_{GS}$  for  $M_2$  must be  $> 1.0$  V.



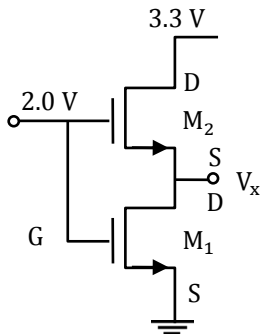
Current through the nMOS transistors can be modeled as

$$I_{DS} = \mu C_{ox} \left( \frac{W}{L} \right) \left( (V_{GS} - V_T) V_{DS} - \frac{1}{2} V_{DS}^2 \right) \quad \text{for } V_{DS} \leq V_{GS} - V_T$$

$$I_{DS} = \mu C_{ox} \left( \frac{W}{L} \right) (V_{GS} - V_T)^2 / 2 \quad \text{for } V_{DS} \geq V_{GS} - V_T$$

The voltage (in volts, accurate the two decimal places) at  $V_x$  is \_\_\_\_\_.

[Ans. \*] Range: 0.41 to 0.435



<b>M<sub>2</sub>:</b>	<b>M<sub>1</sub>:</b>
$V_{DS_2} = 3.3 - V_x$	$V_{GS_1} = 2V$
$V_{GS_2} = 2 - V_x$	$V_{DS_1} = V_x$
$V_{GS_2} > 1$ (Given)	
$\Rightarrow V_x < 1$	

<b>M<sub>2</sub>:</b>	<b>M<sub>1</sub></b>
$V_{DS} > V_{GS} - V_T$	$V_{DS} < V_{GS} - V_T$
$3.3 - V_x > 2 - V_x - 1$	$V_x < 2 - 1$
$\Rightarrow V_{DS} > V_{GS} - V_T$	As $V_x < 1$
Saturation region	Linear region

$$(sat)I_{DS_2} = I_{DS_1}(\text{Linear})$$

$$I_{D_2(sat)} = I_{D_1(sat)}$$

$$\mu_n \text{COX} \left( \frac{\omega}{L} \right)_2 \left[ \frac{(V_{GS_2} - V_T)^2}{2} \right] = \mu_n \text{COX} \left[ \frac{\omega}{L} \right]_1 \left[ (V_{GS_1} - V_T) V_{DS_1} - \frac{V_{DS_1}^2}{2} \right]$$

$$\left[ \frac{\omega}{L} \right]_2 = 2 \left[ \frac{\omega}{L} \right]_1$$

$$\Rightarrow 2 \frac{(V_{GS_2} - V_T)^2}{2} = (V_{GS_1} - V_T)V_{DS_1} - \frac{V_{DS_1}^2}{2}$$

$$(2 - V_x - 1)^2 = (1)(V - x) - \frac{V_x^2}{2}$$

$$V_x^2 + 1 - 2V - x = V_x - \frac{V_x^2}{2}$$

$$\frac{3V_x^2}{2} - 3V_x + 1 = 0$$

$$3V_x^2 - 6V_x + 2 = 0$$

$$V_x = \frac{6 \pm \sqrt{36 - 4(3)(2)}}{2(3)} = 1 \pm \frac{2\sqrt{3}}{2(3)} = 1 \pm \frac{1}{\sqrt{3}}$$

$$\text{As } V_x > 1 \Rightarrow V - x = 1 - \frac{1}{\sqrt{3}} = 0.423 \text{ V}$$

50. A curve passes through the point  $(x = 1, y = 0)$  and satisfies the differential equation

$$\frac{dy}{dx} = \frac{x^2 + y^2}{2y} + \frac{y}{x}. \text{ The equation that describes the curve is}$$

(A)  $\ln\left(1 + \frac{y^2}{x^2}\right) = x - 1$

(B)  $\frac{1}{2} \ln\left(1 + \frac{y^2}{x^2}\right) = x - 1$

(C)  $\ln\left(1 + \frac{y}{x}\right) = x - 1$

(D)  $\frac{1}{2} \ln\left(1 + \frac{y}{x}\right) = x - 1$

[Ans. A]

$$\frac{dy}{dx} = \frac{x^2 + y^2}{2y} + \frac{y}{x}$$

Option (A)

$$\ln\left(1 + \frac{y^2}{x^2}\right) = x - 1$$

$$\frac{1}{\left(1 + \frac{y^2}{x^2}\right)} \left[ \frac{2yy'(x^2) - y^2(2x)}{(x^2)^2} \right] = 1$$

$$\frac{x^2}{(x^2 + y^2)} \frac{2y'x^2y - 2y^2x}{x^4x^2} = 1$$

$$2y'x^2y - 2y^2x = x^2(x^2 + y^2)$$

$$\Rightarrow \frac{dy}{dx} = \frac{x^2 + y^2}{2y} + \frac{y}{x}$$

∴ Option (A) Satisfies

51. A junction is made between p<sup>-</sup>Si with doping density  $N_{A1} = 10^{15} \text{ cm}^{-3}$  and p Si with doping density  $N_{A2} = 10^{17} \text{ cm}^{-3}$ .

Given: Boltzmann constant  $k = 1.38 \times 10^{-23} \text{ J-K}^{-1}$ , electronic charge  $q = 1.6 \times 10^{-19} \text{ C}$ .

Assume 100 % acceptor ionization.

At room temperature ( $T = 300 \text{ K}$ ), the magnitude of the built in potential (In volts, correct to two decimal places) across this junction will be \_\_\_\_\_.

[Ans. \*]Range: 0.11 to 0.13

$$\frac{P^-}{10^{15}} \quad \frac{P}{10^{17}}$$

$$P_2 = P_1 e^{V_o/V_T}$$

Built in potential

$$V_o = V_T \ln\left(\frac{P_2}{P_1}\right)$$

$$= V_T \ln\left(\frac{10^{17}}{10^{15}}\right)$$

$$= 0.259 \ln(100) = 0.119$$

$$V_o = 0.12 \text{ V}$$

52. The state equation and the output equation of a control system are given below

$$\dot{x} = \begin{bmatrix} -4 & -1.5 \\ 4 & 0 \end{bmatrix} x + \begin{bmatrix} 2 \\ 0 \end{bmatrix} u$$

$$y = [1.5 \quad 0.625]x$$

The transfer function representation of the system is

(A)  $\frac{3s + 5}{s^2 + 4s + 6}$

(B)  $\frac{3s - 1.875}{s^2 + 4s + 6}$

(C)  $\frac{4s + 1.5}{s^2 + 4s + 6}$

(D)  $\frac{6s + 5}{s^2 + 4s + 6}$

[Ans. A]

Transfer function =  $C[SI - A]^{-1} \cdot B$

$$(SI - A) = \begin{bmatrix} S & 0 \\ 0 & S \end{bmatrix} - \begin{bmatrix} -4 & -1.5 \\ 4 & 0 \end{bmatrix}$$

$$(SI - A) = \begin{bmatrix} S + 4 & +1.5 \\ -4 & S \end{bmatrix}$$

$$\text{Cof } [SI - A] = \begin{bmatrix} S & +4 \\ -1.5 & S + 4 \end{bmatrix}$$

$$\text{Adj}(SI - A) = \begin{bmatrix} S & -1.5 \\ 4 & S + 4 \end{bmatrix}$$

$$(SI - A)^{-1} = \frac{\text{Adj}(SI - A)}{[SI - A]} = \frac{1}{S^2 + 4s + 6} \begin{bmatrix} S & -1.5 \\ 4 & S + 4 \end{bmatrix}$$

$$[SI - A]^{-1} B = \begin{bmatrix} S & -1.5 \\ 4 & S + 4 \end{bmatrix} \begin{bmatrix} 2 \\ 0 \end{bmatrix} \times \frac{1}{(S^2 + 4S + 6)}$$

$$(SI - A)^{-1} B = \begin{bmatrix} 2S \\ 8 \end{bmatrix} \times \frac{1}{(S^2 + 4S + 6)}$$

$$C[SI - A]^{-1} B \Rightarrow [1.5 \quad 0.625] \begin{bmatrix} 2S \\ 8 \end{bmatrix} \times \frac{1}{(S^2 + 4S + 6)}$$

$$\text{TF} \Rightarrow \frac{3S + 5}{S^2 + 4S + 6}$$

53. A solar cell of area  $1.0\text{cm}^2$ , operating at 1.0 sun intensity, has a short circuit current of 20 mA and an open circuit voltage of 0.65 V. Assuming room temperature operation and thermal equivalent voltage of 25mV, the open circuit voltage (in volts, correct to two decimal places) at 0.2 sun intensity is\_\_\_\_\_.

**[Ans. \*]Range: 0.59 to 0.63**

For solar cell, short circuit current is the maximum current when load is zero

$$I_{SC} \propto \text{Lintensity}$$

$$\frac{I_{SC_2}}{I_{SC_1}} = \frac{0.2}{1} = \frac{1}{5}$$

$$I \approx I_0 e^{\frac{V}{V_T}} \quad V: \text{Open circuit voltage}$$

$$\frac{I_{SC_2}}{I_{SC_1}} = e^{\frac{V_2 - V_1}{V_T}}$$

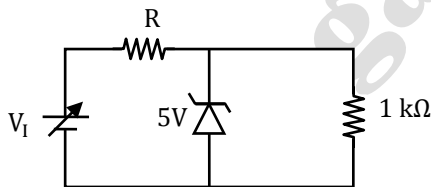
$$-V_T \ln\left(\frac{I_{SC_2}}{I_{SC_1}}\right) = V_1 - V_2$$

$$V_2 = V_1 + V_T \ln\left(\frac{I_{SC_2}}{I_{SC_1}}\right)$$

$$V_2 = 0.65 + V_T \ln\left(\frac{1}{5}\right)$$

$$V_2 = 0.61\text{V}$$

54. The circuit shown in the figure is used to provide regulated voltage (5 V) across the  $1\text{k}\Omega$  resistor. Assume that the Zener diode has a constant reverse breakdown voltage for a current range, smiling from a minimum required Zener current.  $I_{Z_{\min}} = 2\text{ mA}$  to its maximum allowable current. The input voltage  $V_1$  may vary by 5% from its nominal value of 6 V. The resistance of the diode in the breakdown region is negligible.



The value of R and the minimum required power dissipation rating of the diode respectively, are

- (A)  $186\ \Omega$  and 10 mW  
(B)  $100\ \Omega$  and 40 mW  
(C)  $100\ \Omega$  and 10 mW  
(D)  $186\ \Omega$  and 40 mW

**[Ans. B\*]**

$$V_1 = 6\text{V} \pm 5\% = 6\text{V} \pm 0.3\text{V} = 5.7\text{ V to } 6.3\text{ V}$$

$$I_L = \frac{5\text{V}}{1\text{k}\Omega} = 5\text{ mA}$$

$$I_{S(\min)} = I_L + I_{Z(\min)} = 5\text{ mA} + 2\text{ mA} = 7\text{ mA}$$

$$I_S = \frac{V_1 - V_Z}{R}$$

$$I_{S(\min)} = \frac{V_{I(\min)} - V_Z}{R_{\max}} = 7\text{ mA}$$

$$\text{So, } R_{\max} = \frac{5.7 - 5}{7} \text{ k}\Omega = \frac{700}{7} \Omega = 100 \Omega$$

$$\text{When, } R + 100 \Omega, I_{S(\max)} = \frac{6.3 - 5}{100} \text{ A} = 13 \text{ mA}$$

$$I_{Z(\max)} = I_{S(\max)} - I_L = 13 \text{ mA} - 5 \text{ mA} = 8 \text{ mA}$$

$$P_{Z(\min)} = V_Z I_{Z(\max)} = (5 \times 8) \text{ mW} = 40 \text{ mW}$$

55. The distance (in meters) a wave has to propagate in a medium having a skin depth of 0.1 m so that the amplitude of the wave attenuates by 20 dB is

(A) 0.12

(B) 0.23

(C) 0.46

(D) 2.5

**[Ans. B]**

$$E(x) = E_0 e^{-\alpha x}$$

At  $x = \delta$  (Skin depth)

$$E = E_0 e^{-1}$$

$$\alpha \delta = 1$$

$$\delta = \frac{1}{\alpha} \Rightarrow \alpha = \frac{1}{\delta}$$

$$E(x) = E_0 e^{-\frac{x}{\delta}}$$

$$E_1(\text{dB}) = E_0(\text{dB}) - 20$$

$$20 \log E_1 - 20 \log E_0 = -20$$

$$20 \log \left( \frac{E_1}{E_0} \right) = -20$$

$$\Rightarrow E_1 = \frac{E_0}{10} = E_0 e^{-\frac{x}{\delta}}$$

$$e^{\frac{x}{\delta}} = 10$$

$$x = 8 \ln(10)$$

$$x = 0.1 \ln(10)$$

$$x = 0.23 \text{ m}$$

