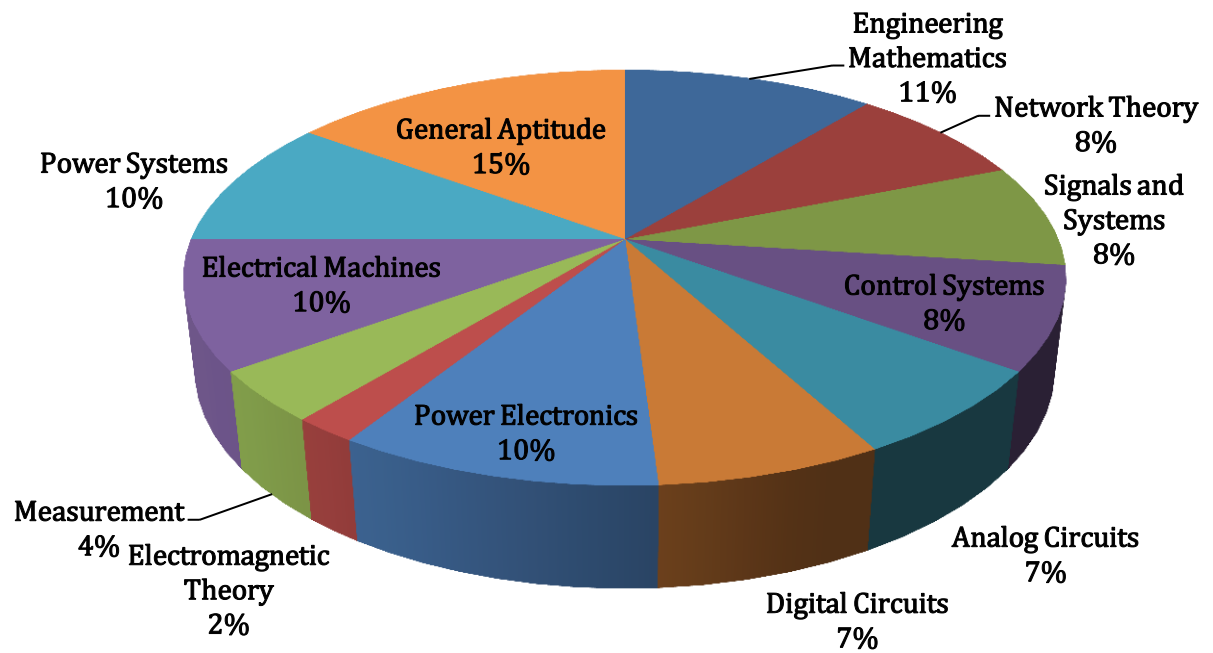


ANALYSIS OF GATE 2018

Electrical Engineering



EE ANALYSIS-2018_10-Feb_Afternoon

SUBJECT	No. of Ques.	Topics Asked in Paper(Memory Based)	Level of Ques.	Total Marks
Engineering Mathematics	1 Marks: 3 2 Marks: 4	Statistics and Probability Calculus; Differential Equations; Complex Variables;	Tough	11
Network Theory	1 Marks: 2 2 Marks: 3	Basic Components and types of circuits; Steady state analysis of AC Circuits; Two Port Networks	Medium	8
Signals and Systems	1 Marks: 2 2 Marks: 3	Linear Time Invariant(LTI) Systems; Fourier Representation of Signals; Z- Transform; Laplace Transform;	Tough	8
Control Systems	1 Marks: 4 2 Marks: 2	Basics Of Control System; Time Domain Analysis; Stability Analysis; Frequency Domain Analysis;	Tough	8
Analog Circuits	1 Marks: 3 2 Marks: 2	Diode Circuits-Analysis and Application; AC & DC Biasing-BJT and FET; Operational Amplifiers	Tough/ Easy	7
Digital Circuits	1 Marks: 1 2 Marks: 3	Boolean Algebra and Karnaugh Maps; Logic Gates; Combinational and Sequential Digital Circuits;	Easy	7
Power Electronics	1 Marks: 2 2 Marks: 4	Power Semiconductor Devices; Choppers; Inverters	Tough	10
Electromagnetic Theory	1 Marks: 2 2 Marks: 0	Electromagnetic Field	Moderate	2
Measurement	1 Marks: 2 2 Marks: 1	Basics of Measurements and Error Analysis; Electronic Measuring Instruments	Moderate	4
Electrical Machines	1 Marks: 2 2 Marks: 4	Transformer; Three Phase Induction Motors; D.C. Machine; Synchronous Machine;	Tough/Easy	10
Power Systems	1 Marks: 2 2 Marks: 4	Transmission & Distribution; Economics of Power Generation; Symmetrical Components & Faults Calculations; Power System Stability;	Tough	10
General Aptitude	1 Marks: 5 2 Marks: 5	Probability; Time Distance; Permutation	Easy	15
Total	65			100
Faculty Feedback	Majority of the question were concept based. Paper was Slightly Tougher, but very Lengthy			

GATE 2018 Examination

Electrical Engineering

Test Date: 10-Feb-2018

Test Time: 2:00 PM 5:00 PM

Subject Name: Electrical Engineering

General Aptitude

Q.1 - Q.5 Carry One Mark each.

1. Function $F(a, b)$ and $G(a, b)$ are defined as follows
 $F(a, b) = (a - b)^2$ and $G(a, b) = |a - b|$, where $|x|$ represents the absolute value of x
What would be the value of $G(F(1,3), G(1,3))$?
- (A) 2 (B) 4
(C) 6 (D) 36
- [Ans. A*]**
 $F(a, b) = (a - b)^2$
 $G(a, b) = |a - b|$
 $G(F(1,3), G(1,3)) = G((1 - 3)^2, |1 - 3|)$
 $= G(4, 2)$
 $= |4 - 2| = 2$
2. "A common misconception among writers is that sentence structure mirrors thought, the more _____ the structure, the more complicated the ideas"
- (A) detailed (B) Simple
(C) Clear (D) convoluted
- [Ans. D]**
3. Since you have gone off the _____, the _____ sand is likely to damage the car.
- (A) Course, coarse (B) Course, course
(C) Coarse, course (D) Coarse, coarse
- [Ans. A*]**
- Going off the course - not following the intended route.
 - Coarse sand - harsh in texture
4. The three roots of the equation $f(x) = 0$ are $x = \{-2, 0, 3\}$. What is the three value of x for which $f(x - 3) = 0$?
- (A) $-5, -3, 0$ (B) $-2, 0, 3$
(C) $0, 6, 8$ (D) $1, 3, 6$
- [Ans. D*]**

$$f(x) = 0$$

$$x = (-2, 0, 3)$$

$$f(-2) = 0, f(0) = 0, f(3) = 0$$

$$f(x - 3) = 0$$

$$1, 3, 6$$

$$f(1 - 3) = f(-2) = 0$$

$$f(3 - 3) = f(0) = 0$$

$$f(6 - 3)f(3) = 0$$

5. For what values of k given below is $\frac{(k+2)^2}{k-3}$ an integer
- (A) 4,8,18 (B) 4,10,16
(C) 4,8,28 (D) 8,26,28

[Ans. C*]

$$\left. \begin{aligned} K = 4 &\Rightarrow \frac{(4+2)^2}{4-3} = 36 \\ K = 8 &\Rightarrow \frac{(8+2)^2}{8-3} = \frac{100}{5} = 20 \\ K = 28 &\Rightarrow \frac{(28+2)^2}{28-3} = \frac{900}{25} = 36 \end{aligned} \right\} \text{Integers}$$

So, option (C) = 4, 8, 28

Q.6 - Q.10 Carry Two Mark each.

6. "An e-mail password must contain three characters. The password has to contain one numeral from 0 to 9, one upper case and one lower case character from the English alphabet. How many distinct passwords are possible?
- (A) 6,760 (B) 13,520
(C) 40,560 (D) 1,05,456

[Ans. C]

Numeral can be selected in 10 ways (0 - 9). Each of upper case and lower case alphabets can be done in 26 ways each. All three chosen (1 numeral and 2 alphabets) can be arrange in 3! ways. So, total number of ways will be $10 \times 26 \times 26 \times 3! = 40560$ ways

7. A class of twelve children has two boys more than girls. A group of three children are randomly picked from this class to accompany the teacher on a field trip. What is the probability that the group accompanying the teacher contains more girls than boys?
- (A) 0 (B) $\frac{325}{864}$
(C) $\frac{525}{864}$ (D) $\frac{5}{12}$

[Ans. B *]

$$B + G = 12$$

$$B = G + 2$$

$$\Rightarrow B = 7$$

$$G = 5$$

7 Boys and 5 Girls are there is 12 students. Among 3 students selected boys have to be more than girls.

So only two cases arise.

G	B
2	1
3	0

8. P, Q, R and S crossed a lake in a boat that can hold a maximum of two persons, with only one set of oars. The following additional facts are available.
- (i) The boat held two persons on each of three forward trips across the lake and one person on each of the two return trips.
 - (ii) P is unable to row when someone else is in the boat
 - (iii) Q is unable to row with anyone else except R
 - (iv) Each person rowed for at least one trip
 - (v) Only one person can row during a trip

Who rowed twice?

- (A) P (B) Q
(C) R (D) S

[Ans. C*]

- (i) Q and R moves first.
In forward trip Q rowed.
In return trip R rowed.
- (ii) P and R moves in second trip.
R rowed in forward trip.
P rowed in return trip.
- (iii) P and S moves in last trip.
S rowed in forward trip.
R rowed twice.

9. In a certain code, AMCF is written as EQGJ and NKUF written as ROYJ. How will DHLP be written in that code?
- (A) RSTN (B) TLPH
(C) HLPT (D) XSVR

[Ans. C]

10. A designer uses marbles of four different colours for his design. The cost of the each marble is the same irrespective of the colour. The table shows the percentage of marbles of each colour used in the current design. The cost of each marble increased by 25%. Therefore, the designer decided to reduce equal numbers of the marbles of each colour to keep the total cost unchanged. What is the percentage of blue marbles in the new design?

Blue	Black	Red	Yellow
40 %	25 %	20 %	15 %

- (A) 35.75 (B) 40.25
(C) 43.75 (D) 46.25

[Ans. C*]

Assume number of marbles = 100

Cost of marbles increased = 25

New cost = Rs. 125

Number of marbles in Rs. 100 = $\frac{100}{125} \times 100 = 80$ marbles

Now, $(40 - x) + (25 - x) + (20 - x) + (15 - x) = 80$

$100 - 4x = 80$

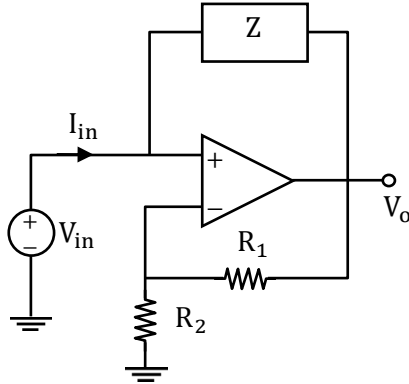
$x = 5$

% Blue marbles in new design = $\frac{(40 - 5)}{80} \times 100$
 $= 35 \times \frac{5}{4} = 43.75\%$

Technical

Q.1 - Q.25 Carry One Mark each.

1. The op-amp shown in the figure is ideal. The input impedance $\frac{V_{in}}{I_{in}}$ is given by



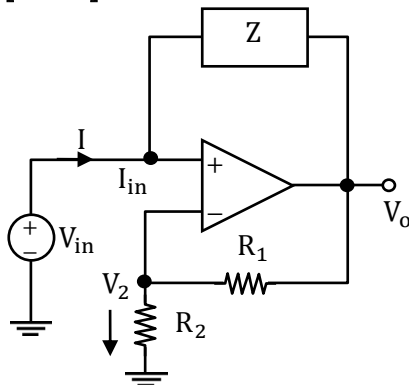
(A) $Z \frac{R_1}{R_2}$

(B) $-Z \frac{R_2}{R_1}$

(C) Z

(D) $-Z \frac{R_1}{R_1+R_2}$

[Ans. B]



$$I_{in} = \frac{V_{in} - V_o}{Z} \dots \dots \textcircled{1}$$

Now, due to virtual short circuit,

$$V_2 = V_{in}$$

$$\frac{V_o - V_2}{R_1} = \frac{V_2}{R_2} \Rightarrow \frac{V_o - V_{in}}{R_1} = \frac{V_{in}}{R_2}$$

$$\Rightarrow \frac{V_o}{R_1} = V_{in} \left[\frac{1}{R_1} + \frac{1}{R_2} \right] = V_{in} \left[\frac{R_1 + R_2}{R_1 R_2} \right]$$

$$\Rightarrow V_o = \left[\frac{R_1 + R_2}{R_2} \right] V_{in} \dots \dots \textcircled{2}$$

Substituting in $\textcircled{1}$

$$I_{in} = \frac{V_{in} - \left[\frac{R_1 + R_2}{R_2} \right] V_{in}}{Z}$$

$$I_{in} = \left[\frac{R_2 - R_1 - R_2}{R_2 Z} \right] V_{in}; \Rightarrow \frac{V_{in}}{I_{in}} = - \frac{R_2 Z}{R_1}$$

2. Two wattmeter methods is used for measurement of power in a balanced three-phase load supplied form a balanced three-phase system. If one of the wattmeter's reads half of the other (both positive).Then the power factor of the load is ?
 (A) 0.532 (B) 0.632
 (C) 0.707 (D) 0.866

[Ans. D]

Two wattmeter methods gives,

$$W_1 = V_L I_L \cos(\phi - 30^\circ)$$

$$W_2 = V_L I_L \cos(\phi + 30^\circ)$$

Now, We know $\cos(\phi - 30^\circ) > \cos(\phi + 30^\circ)$

$$\therefore W_1 > W_2$$

$$\text{Given, } \frac{W_1}{2} = W_2$$

$$\Rightarrow \frac{1}{2} \cos(\phi - 30^\circ) = \cos(\phi + 30^\circ)$$

If $\phi = 30^\circ$

$$\frac{1}{2} \times \cos 0^\circ = \cos 60^\circ$$

$$\frac{1}{2} \times 1 = \frac{1}{2}$$

$$\therefore \phi = 30^\circ$$

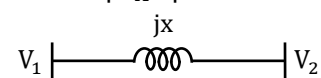
$$\therefore \text{Power factor} = \cos \phi = \cos 30^\circ = 0.866$$

3. Consider a lossy transmission line with V_1 and V_2 as the sending and receiving end voltages respectively Z and X are the series impedance and reactance of the line, respectively. The steady-state stability limit for the transmission line will be
 (A) Greater than $\left| \frac{V_1 V_2}{X} \right|$ (B) Less than $\left| \frac{V_1 V_2}{X} \right|$
 (C) Equal to $\left| \frac{V_1 V_2}{X} \right|$ (D) Equal to $\left| \frac{V_1 V_2}{Z} \right|$

[Ans. B*]

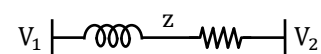
With only x

$$P_{\max} = \left| \frac{V_1 V_2}{X} \right|$$



With Lossy Tr. Line

$$P = \left| \frac{V_1 V_2}{Z} \right| \cos(\beta - \delta) - \left| \frac{AV_2^2}{Z} \right| \cos(\beta - \alpha)$$



$$\therefore \text{With Lossy line } P_{\max} < \left| \frac{V_1 V_2}{X} \right|$$

4. A 1000×1000 bus admittance matrix for an electric power system has 8000 non-zero elements. The minimum number of branches (transmission lines and transformers) in this system is _____ (up to 2 decimal places).

[Ans. *] Range: 3500.0 to 3500.0

Total non-zero element = 8000

Total diagonal element = 1000

$$\text{Minimum number of branches} = \frac{7000}{2} = 3500$$

5. Let f be a real-valued function of a real variable defined as $f(x) = x^2$ for $x \geq 0$, and $f(x) = -x^2$ for $x < 0$. Which one of the following statements is true?
 (A) $f(x)$ is discontinuous at $x = 0$.
 (B) $f(x)$ is continuous but not differentiable at $x = 0$.
 (C) $f(x)$ is differentiable but its first derivative is not continuous at $x = 0$.
 (D) $f(x)$ is differentiable but its first derivative is not differentiable at $x = 0$.

[Ans. D]

$$f(x) = \begin{cases} x^2 & x \geq 0 \\ -x^2 & x < 0 \end{cases}$$

$$f'(x) = \begin{cases} 2x & x \geq 0 \\ -2x & x < 0 \end{cases}$$

$$f''(x) = \begin{cases} 2 & x \geq 0 \\ -2 & x < 0 \end{cases}$$

The first derivation of $f(x)$ is not derivable at $x = 0$

6. The value of the directional derivative of the function $\phi(x, y, z) = xy^2 + yz^2 + zx^2$ at the point $(2, -1, 1)$ in the direction of the vector $p = i + 2j + 2k$ is
 (A) 1 (B) 0.95
 (C) 0.93 (D) 0.9

[Ans. A]

$$\phi = xy^2 + yz^2 + zx^2$$

$$\nabla\phi = \bar{i} \frac{\partial\phi}{\partial x} + \bar{j} \frac{\partial\phi}{\partial y} + \bar{k} \frac{\partial\phi}{\partial z}$$

$$= \bar{i}(y^2 + 2xz) + \bar{j}(2xy + z^2) + \bar{k}(2yz + x^2)$$

$$\nabla\phi_{(2,-1,1)} = \bar{i}(1 + 4) + \bar{j}(-4 + 1) + \bar{k}(-2 + 4)$$

$$= 5\bar{i} - 3\bar{j} + 2\bar{k}$$

$$\bar{P} = \bar{i} + 2\bar{j} + 2\bar{k}$$

$$|\bar{P}| = \sqrt{1 + 4 + 4} = 3$$

The directional derivative of $\phi(x, y, z)$ at $(2, -1, 1)$ in the direction of \bar{P} is $\nabla\phi_{\text{at } P} \cdot \frac{\bar{P}}{|\bar{P}|}$

$$= (5\bar{i} - 3\bar{j} + 2\bar{k}) \left(\frac{\bar{i} + 2\bar{j} + 2\bar{k}}{3} \right)$$

$$= \frac{5 - 6 + 4}{3} = 1$$

7. The graph of a network has 8 nodes and 5 independent loops. The number of branches of the graph is
 (A) 11 (B) 12
 (C) 13 (D) 14

[Ans .B]

$$\text{Loops } b - (N - 1)$$

$$5 = b - (8 - 1)$$

$$5 = b - 7$$

$$b = 12$$

8. Match the transfer function of the second-order system with the nature of the system

Transfer functions	Nature of system
P. $\frac{15}{s^2+5s+15}$	I. Over-damped
Q. $\frac{25}{s^2+10s+25}$	II. Critically -damped
R. $\frac{35}{s^2+18s+35}$	III. Under damped

(A) P-I, Q-II, R-III

(B) P-II, Q- I, R- III

(C) P-III, Q-II, R-I

(D) P-III, Q-I, R-II

[Ans. C]

$$P = \frac{15}{s^2 + 5s + 15} = \frac{\omega_n^2}{s^2 + 2\xi\omega_n s + \omega_n^2}$$

$$\omega_n = \sqrt{15} = 3.67$$

$$2\xi\omega_n = 5$$

$$\xi = \frac{2.5}{3.67} = 0.64; \text{ **Under damped**}$$

$$Q = \frac{25}{s^2 + 10s + 25} = \frac{\omega_n^2}{s^2 + 2\xi\omega_n s + \omega_n^2}$$

$$\omega_n = 5; 2\xi\omega_n = 10 \rightarrow \xi = 1; \text{ **Critically damped**}$$

$$R = \frac{35}{s^2 + 18s + 35} = \frac{\omega_n^2}{s^2 + 2\xi\omega_n s + \omega_n^2}$$

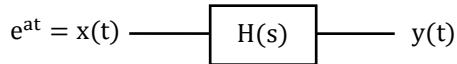
$$\omega_n = \sqrt{35} = 5.91; 2\xi\omega_n = 18$$

$$\xi = \frac{9}{5.91} = 1.52; \text{ **Over damped**}$$

9. A continuous-time input signal $x(t)$ is an eigen function of an LTI system. If the output is
 (A) $k x(t)$. where k is an eigenvalue
 (B) $k e^{j\omega t} x(t)$, where k is an eigenvalue and $e^{j\omega t}$ is a complex exponential signal
 (C) $x(t)e^{j\omega t}$, where $e^{j\omega t}$ is a complex exponential signal
 (D) $k H(\omega)$, where k is an eigenvalue and $H(\omega)$ is a frequency response of the system

[Ans. A*]

Eigen Function is a type of input for which output is constant times of input i.e.,



Where $x(t)$ = system input = Eigen function

$H(s)$ = Transfer function of system

$y(t)$ = System output

Here, $y(t) = H(s)|_{s=a} \cdot e^{at}$

= $K \cdot x(t)$

Where, K = Eigen-value = $H(s)|_{s=a}$

$x(t)$ = Eigen – function input = $H(s)|_{s=a}$

10. Consider a non-singular 2×2 square matrix A . If $\text{trace}(A) = 4$ and $\text{trace}(A^2) = 5$. The determinant of the matrix A is _____ (up to 1 decimal place).

[Ans. *] Range: 5.5 to 5.5

A is 2×2 matrix

$\text{Tr } A = 4$

$\lambda_1 + \lambda_2 = 4$

$\text{tr}(A^2) = 5$

$\lambda_1^2 + \lambda_2^2 = 5$

$(\lambda_1 + \lambda_2)^2 = \lambda_1^2 + \lambda_2^2 + 2\lambda_1\lambda_2$

$16 = 5 + 2\lambda_1\lambda_2$

$2\lambda_1\lambda_2 = 11$

$\lambda_1\lambda_2 = \frac{11}{2}$

$|A| = \frac{11}{2} = 5.5$

11. The positive, negative and zero sequence impedances of a 125 MVA, three-phase, 15.5 kV, star-grounded, 50 Hz generator are $j0.1$ pu, $j0.05$ pu and $j0.01$ pu respectively on the machine rating base. The machine is unloaded and working at the rated terminal voltage. If the grounding impedance of the generator is $j0.01$ pu, then the magnitude of fault current for a b-phase to ground fault (in kA) is _____ (up to 2 decimal places).

[Ans. *] Range: 73.30 to 73.70

Given $z_1 = 0.1j$, $z_2 = 0.05j$, $z_0 = 0.01j$

$z_s = 0.01$

We know that for L-G fault

$I_s = 3 \cdot I_{as}$

$$= \frac{3 \times 1}{z_1 + z_2 + z_0 + 3z_s}$$

$$= \frac{3 \times 1}{(0.1 + 0.05 + 0.01 + 0.01)j}$$

$$= \frac{3}{0.19} = 15.79 \angle -90^\circ$$

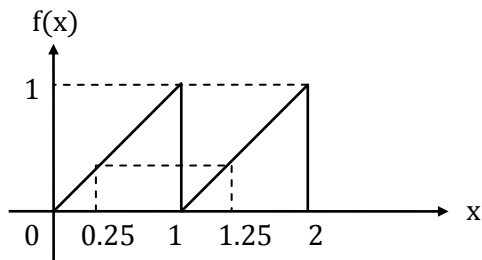
$|I_f| = 15.79$ pu

$$I_B = \frac{125 \text{ MVA}}{\sqrt{3} \times 15.5 \text{ kV}} = 4.66 \text{ kA}$$

$$I_s = 15.79 \times I_B = 15.79 \times 4.66 \text{ kA} = 73.5 \text{ kA}$$

12. Let f be a real-valued function of a real variable defined as $f(x) = x - [x]$, where $[x]$ denotes the largest integer less than or equal to x . The value of $\int_{0.25}^{1.25} f(x) dx$ is _____ [up to 2 decimal places]

[Ans. *] Range: 0.49 to 0.51



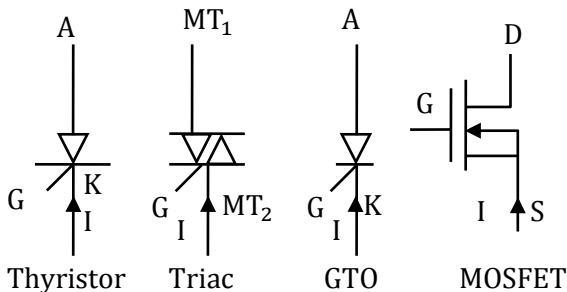
$$\int_{0.25}^{1.25} f(x) dx \Rightarrow \int_{\frac{1}{4}}^1 (x) dx + \int_1^{\frac{5}{4}} (x - 1) dx$$

$$\Rightarrow \left(\frac{x^2}{2}\right)_{\frac{1}{4}}^1 + \left(\frac{x^2}{2} - x\right)_1^{\frac{5}{4}}$$

$$\Rightarrow \left(\frac{1}{2} - \frac{1}{32}\right) + \left[\left(\frac{25}{32} - \frac{5}{4}\right) - \left(\frac{1}{2} - 1\right)\right]$$

$$\Rightarrow \frac{15}{32} + \frac{1}{32} = \frac{16}{32} = \frac{1}{2} = 0.5$$

13. Four power semiconductor devices are shown in the figure along with their relevant terminals. The device(s) that can carry dc current continuously in the direction shown when gated appropriately is (are)



- (A) Triac only
(B) Triac and MOSFET
(C) Triac and GTO
(D) Thyristor and Triac

[Ans. B]

14. Consider a unity feedback system with forward transfer function given by $G(s)H(s) = \frac{1}{(s+1)(s+2)}$. The steady state error in the output of the system for a unit -step input is _____ (up to 2 decimal places)

[Ans. *]Range: 0.65 to 0.69

$$e_{ss} = \frac{1}{1 + k_p} \quad k_p \underset{s \rightarrow 0}{\text{lt}} GH = \frac{1}{2}$$

$$\therefore e_{ss} = \frac{1}{1 + \frac{1}{2}} = \frac{2}{3} = 0.67$$

15. The value of the integral $\oint_C \frac{z+1}{z^2-4} dz$ in counter clockwise direction around a circle C of radius 1 with center at the point $z = -2$ is

(A) $\pi i/2$

(B) $2\pi i$

(C) $-\pi i/2$

(D) $-2\pi i$

[Ans. A]

$$\int \frac{z+1}{z^2-4} dz$$

$$\int \frac{z+1}{(z-2)(z+2)} dz$$

$$\int \frac{\left(\frac{z+1}{z-2}\right)}{(z+2)} dz$$

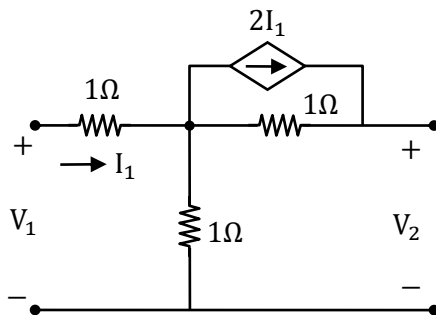
$$\text{Where, } f(z) = \frac{z+1}{z-2}$$

$$= 2\pi i f(-2)$$

$$= 2\pi i \left(\frac{-2+1}{-2-2}\right)$$

$$= 2\pi i \left(\frac{-1}{-4}\right) = \frac{\pi i}{2}$$

16. In a two port network shown, the h_{11} parameter (where, $h_{11} = \frac{V_1}{I_1}$, When $V_2 = 0$) in ohms is _____ (up to 2 decimal places)



[Ans. *]Range: 0.45 to 0.55

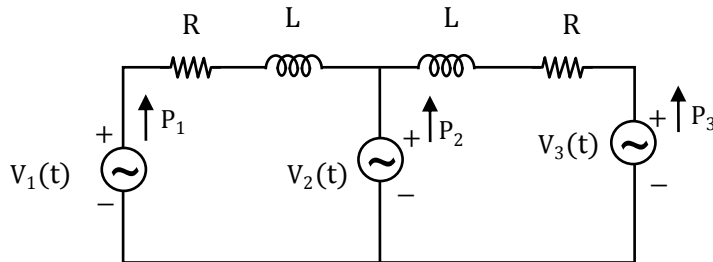
Let us put 1A test source $I_1 = 1A$, $\therefore 2I_1 = 2A$

Nodal equation

$$\begin{aligned} \text{At } A \Rightarrow -1 + 2 + \frac{V_A}{1} + \frac{V_A}{1} &= 0 \\ \therefore V_A &= -\frac{1}{2} = -0.5 \\ \therefore V_1 &= V_A + 1 = 0.5V \\ \therefore h_{11} &= \frac{V_1}{I_1} = \frac{0.5}{1} = \frac{1}{2} = 0.5 \end{aligned}$$

17. In the figure, the voltage are $V_1(t) = 100 \cos \omega t$, $V_2(t) = 100 \cos \left(\omega t + \frac{\pi}{18} \right)$ and $V_3(t) = 100 \cos \left(\omega t + \frac{\pi}{36} \right)$.

The circuit is in sinusoidal steady state, and $R \ll \omega L$. P_1, P_2 and P_3 are the average power outputs. Which one of the following statements is true?



- (A) $P_1 = P_2 = P_3 = 0$ (B) $P_1 < 0; P_2 > 0; P_3 > 0$
(C) $P_1 < 0; P_2 > 0; P_3 < 0$ (D) $P_1 > 0; P_2 < 0; P_3 > 0$

[Ans. C]

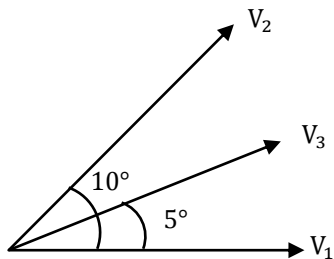
$$V_2: \frac{\pi}{18} = \frac{180^\circ}{18} = 10^\circ$$

$$V_3: \frac{\pi}{36} = \frac{180^\circ}{36} = 5^\circ$$

V_2 leads V_1 and V_3 .

So, V_2 is a source, V_1 and V_3 are absorbing Hence.,

$$P_2 > 0, P_1, P_3 < 0$$



18. The series impedance matrix of a short three-phase transmission line in phase coordinates

is $\begin{bmatrix} Z_s & Z_m & Z_m \\ Z_m & Z_s & Z_m \\ Z_m & Z_m & Z_s \end{bmatrix}$. If the positive sequence impedance is $(1 + j 10) \Omega$ and the zero sequence

is $(4 + j 31) \Omega$, then the imaginary part of Z_m (in Ω) is (up to 2 decimal places).

[Ans. *] Range: 7 to 7

$$Z_1 = (1 + j10)\Omega, Z_0 = (G + j31)\Omega$$

We know that from symmetrical components

$$Z_0 = Z_s + 2Z_m$$

$$Z_1 = Z_s - Z_m$$

$$\Rightarrow Z_s + 2Z_m = 1 - j10$$

$$Z_s - 0Z_m = 4 + j31$$

$$\hline 3Z_m = -3 - j21$$

$$Z_m = -1 - j7$$

So, imaginary part of $Z_m = 7$

19. A single-phase 100 kVA, 1000V/100 V, 50 Hz transformer has a voltage drop 5% across its series impedance at full load, of this 3% is due to resistance. The percentage regulation of the transformer at full load with 0.8 lagging power factor is

(A) 4.8

(B) 6.8

(C) 8.8

(D) 10.8

[Ans. A]

$$\text{Percent voltage regulation} = \left\{ \frac{I_2 R_{02}}{E_2} \times 100 \right\} \cos \phi_2 \pm \left(\frac{I_2 X_{02}}{E_2} \times 100 \right) \sin \phi_2$$

$$\%V.R. = V_r \cos \phi_2 \pm V_x \sin \phi_2$$

(where, '+' lag pf. and '-' lead pf)

At full load;

Given; $V_r = 3\%$

Impedance drop, $V_z = 5\%$

$$\therefore \text{Reluctance drop, } V_x = \sqrt{5^2 - 3^2} = 4\%$$

Voltage regulation at full load at 0.8 p.f. lagging

$$V.R. = 3(0.8) + 4(0.6)$$

$$= 2.4 + 2.4 = 4.8\%$$

20. In a salient pole synchronous motor, the developed reluctance torque attains the maximum value when the load angle in electrical degrees is

(A) 0

(B) 45

(C) 60

(D) 90

[Ans. B]

We know that total power developed in salient pole synchronous motor is

$$P_{\text{total}} = \underbrace{\frac{E\delta V_t}{X_t} \sin \delta}_{\text{Electromagnetic Power}} + \underbrace{\frac{V_t^2}{2} \left(\frac{1}{x_q} - \frac{1}{x_d} \right) \sin 2\delta}_{\text{Reluctance Power}}$$

So, reluctance power would minimum when

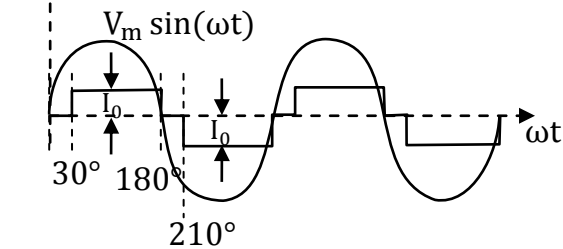
$$\sin 2\delta = 1$$

$$2\delta = 90^\circ$$

$$\delta = 45^\circ$$

21. The waveform of the current drawn by a semi-converter from a sinusoidal AC voltage source is shown in the figure. If $I_0 = 20$ A. The rms value of fundamental component of the current is _____ A. (up to 2 decimal places).
voltage and current

Voltage and current



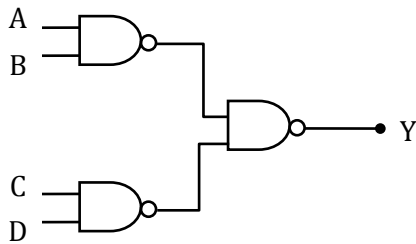
[Ans. *] Range: 16.90 to 17.70*

$$i_{s1} = \frac{4I_0}{\pi} \cos \frac{\alpha}{2}$$

$$i_{s1(\text{rms})} = \frac{2\sqrt{2}}{\pi} I_0 \cdot \cos \frac{\alpha}{2}$$

$$= \frac{2\sqrt{2}}{\pi} \times 20 \times \cos \left(\frac{30^\circ}{2} \right) = 17.39 \text{ A}$$

22. In the logic circuit shown in the figure, Y is given by



(A) $Y=ABCD$

(B) $Y=(A+B)(C+D)$

(C) $Y= A+B+C+D$

(D) $Y=AB+CD$

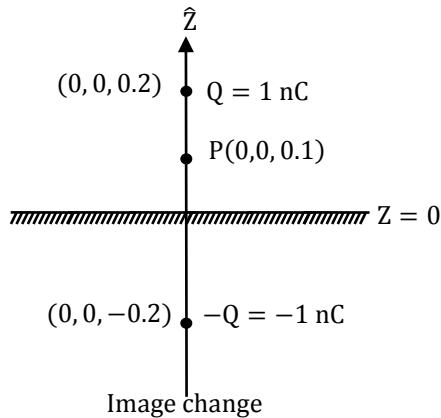
[Ans. D]

$$f = \overline{\overline{AB}} \cdot \overline{\overline{CD}}$$

$$= \overline{\overline{AB}} + \overline{\overline{CD}} = AB + CD$$

23. A positive charge of 1 nC is placed at (0, 0, 0.2) where all dimensions are in metres. Consider the x-y plane to be a conducting ground plane. Take $\epsilon_0 = 8.85 \times 10^{-12}$ F/m. The z- component of the E field at (0,0,0.1) is closest to
(A) 899.18 V/m (B) -899.18 V/m
(C) 999.09 V/m (D) -999.09 V/m

[Ans. D]



Net electric field at point P due to charge Q is

$$\vec{E}_{12} = \frac{Q\vec{R}_{12}}{4\pi\epsilon_0|\vec{R}_{12}|^3}$$

$$\begin{aligned} \text{Total, } \vec{E}_{12} &= \frac{1 \times 10^{-9}(-0.1\hat{a}_z)}{4\pi(8.854 \times 10^{-12})(0.1)^3} + \frac{(-1 \times 10^{-9})((0.3)\hat{a}_z)}{4\pi(8.854 \times 10^{-12})(0.3)^3} \\ &= \left[\frac{-10^5}{4\pi(8.854)} - \frac{10^5}{4\pi(8.854)9} \right] \hat{a}_z \\ &= (-898.774 - 99.863)\hat{a}_z \cong -999.09\hat{a}_z \text{ V/m} \end{aligned}$$

24. A separately excited dc motor has an armature resistance $R_a = 0.05\Omega$. The field excitation is kept constant. At an armature voltage of 100 V, the motor produces a torque of 500 Nm at zero speed. Neglecting all mechanical losses, the no-load speed of the motor (in radian/s) for an armature voltage of 150 V is _____ (up to 2 decimal places).

[Ans. *] Range: 600 to 600*

Armature resistance (R_a) = 0.05Ω

Field excitation is kept constant, so $\phi = \text{constant}$.

Armature voltage (V_t) = 100 V

At zero speed, (τ_0) = 500 N-m.

At zero speed, developed back emf = 0

$$\text{So, armature current}(I_{a0}) = \frac{V_t}{R_a} = \frac{100}{0.05} = 2000\text{A}$$

$$\therefore \tau_0 = K\phi I_{a0}$$

$$\frac{500}{2000} = k\phi$$

$$k\phi = 0.25 \dots \dots \textcircled{1}$$

At no load, as losses are neglected, so developed torque will be zero.

$$\text{From } \tau = k\phi I_a$$

$$I_a = 0$$

The developed back emf,

$$(E_b) = V_t = 150 \text{ V}$$

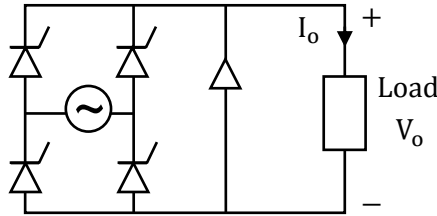
Again, we know that,

$$E_b = k\phi\omega_m$$

$$\text{So, } \omega_m = \frac{E_b}{k\phi} = \frac{150}{0.25}$$

$$\omega_m = 600 \text{ rad/sec}$$

25. A single phase fully controlled rectifier is supplying a load with an anti-parallel diode as shown in the figure. All switches and diodes are ideal. Which one of the following is true for instantaneous load voltage and current?

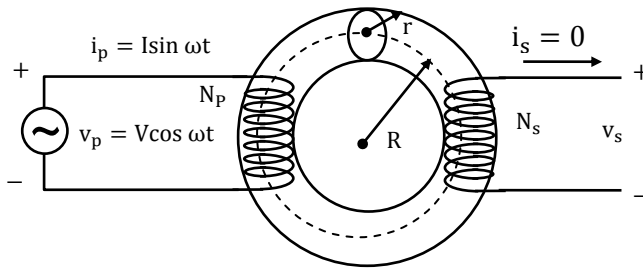


- (A) $V_o \geq 0, I_o < 0$ (B) $V_o < 0, I_o < 0$
(C) $V_o \geq 0, I_o \geq 0$ (D) $V_o < 0, I_o \geq 0$

[Ans. C]

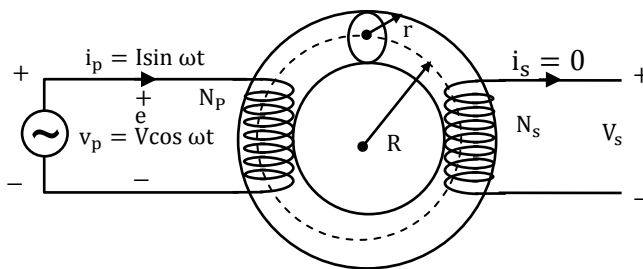
Q.26 - Q.55 Carry One Mark each.

26. A transformer with toroidal core of permeability μ is shown in the figure. Assuming uniform flux density across the circular core cross-section of radius $r \ll R$, and neglecting any leakage flux, the best estimate for the mean radius R is



- (A) $\frac{\mu V r^2 N_p^2 \omega}{I}$ (B) $\frac{\mu I r^2 N_p N_s \omega}{V}$
(C) $\frac{\mu V r^2 N_p^2 \omega}{2I}$ (D) $\frac{\mu I r^2 N_p^2 \omega}{2V}$

[Ans. D*]



Since we know reluctance of core

$$R = \frac{l_c}{\mu a}$$

Here, l_c = Mean core length: a = Area of core

$$\text{Reluctance, } R = \frac{2\pi R}{\mu \times \pi r^2} = \frac{2R}{\mu r^2} \quad [\text{Where, } R = \text{Mean radius of core}]$$

$$\text{or } R = \frac{R \cdot \mu r^2}{2}$$

$$\text{and } R = \frac{\text{mmf}}{\text{flux}} = \frac{N_p I_p}{\phi}$$

$$\text{Here, } \phi = -\frac{1}{N_p} \int_0^t e dt \quad [\text{Where } e \text{ is primary generated voltage}]$$

$$\text{or, } \phi = -\frac{1}{N_p} \int_0^t -(V_p) dt \quad [:: \text{since, } V_p = -e]$$

$$\phi = \frac{1}{N_p} \int_0^t V \cos \omega t dt = \frac{V}{\omega N_p} \sin \omega t$$

$$\text{Now reluctance, } R = \frac{N_p \cdot I \sin \omega t}{\frac{V}{\omega N_p} \sin \omega t} = \frac{N_p^2 \cdot \omega I}{V}$$

$$\text{So mean radius } R = \frac{N_p^2 \times \omega I \times \mu r^2}{2V}$$

27. Consider a system governed by the following equations

$$\frac{dx_1(t)}{dt} = x_2(t) - x_1(t)$$

$$\frac{dx_2(t)}{dt} = x_1(t) - x_2(t)$$

The initial conditions are such that $x_1(0) < x_2(0) < \infty$. Let $x_{1f} = \lim_{t \rightarrow \infty} x_1(t)$ and $x_{2f} = \lim_{t \rightarrow \infty} x_2(t)$. Which one of the following is true?

- (A) $x_{1f} < x_{2f} < \infty$ (B) $x_{2f} < x_{1f} < \infty$
(C) $x_{1f} = x_{2f} < \infty$ (D) $x_{1f} = x_{2f} = \infty$

[Ans. C*]

$$(D + 1)x_1 = x_2 \dots \dots \textcircled{1}$$

$$\Rightarrow (D + 1)x_1 - x_2 = 0$$

$$(D + 1)x_2 = x_1 \dots \dots \textcircled{2}$$

$$\Rightarrow -x_1 + (D + 1)x_2 = 0$$

$$x_{1f} = \lim_{t \rightarrow \infty} x_1(t)$$

$$x_{2f} = \lim_{t \rightarrow \infty} x_2(t)$$

putting $\textcircled{2}$ in $\textcircled{1}$ we get

$$((D + 1)^2 - 1)x_2 = 0$$

$$D^2 + 2D = 0$$

$$D(D + 2) = 0 \quad \therefore x_2 = C_1 + C_2 e^{-2t} \dots \textcircled{1}$$

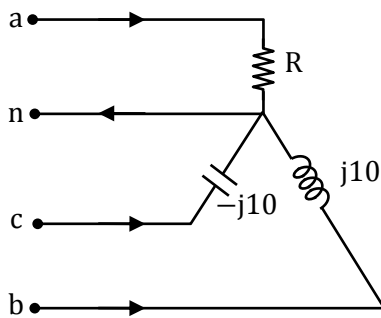
$$D = 0, -2$$

$$x_{2f} = \lim_{t \rightarrow \infty} C_1 + C_2 e^{-2t} = C_1$$

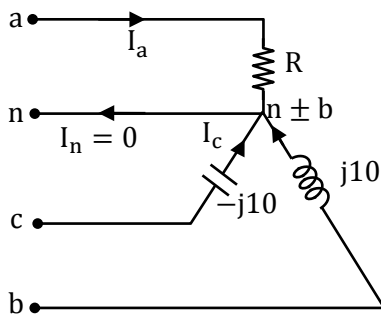
$$(D + 1)x_1 - (D + 1)x_2 = 0$$

$$\begin{aligned}
 -x_1 + (D + 1)x_2 &= 0 \\
 ((D + 1)^2 - 1)x_1 &= 0 \\
 (D_2 + 2D)x_1 &= 0 \\
 \therefore x_{1f} &= \lim_{t \rightarrow \infty} C_1 + C_2 e^{-2t} = C_1 \\
 \therefore x_1 &= C_1 + C_2 e^{-et}
 \end{aligned}$$

28. A three-phase load is connected to a three-phase balanced supply as shown in the figure. If $V_{an} = 100\angle 0^\circ$ V, $V_{bn} = 100\angle -120^\circ$ V, and $V_{cn} = 100\angle -240^\circ$ V (angles are considered positive in the anti-clockwise direction), the value of R for zero current in the neutral wire is _____ Ω (up to 2 decimal places).



[Ans. *]Range: 5.70 to 5.85



$$V_{an} = 100\angle 0^\circ$$

$$V_{bn} = 100\angle -120^\circ$$

$$V_{cn} = 100\angle -210^\circ$$

$$\text{Given } I_n = 0$$

$$I_a + I_b + I_c = 0$$

$$\frac{V_{an}}{R} + \frac{V_{bn}}{j10} + \frac{V_{cn}}{-j10} + \frac{100\angle -210^\circ}{10\angle -90^\circ} = 0$$

$$\frac{100\angle 0^\circ}{R} + 10\angle -210^\circ + 10\angle -150^\circ = 0$$

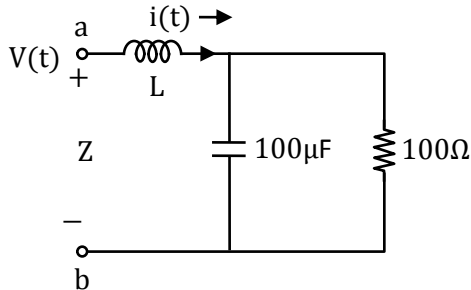
$$\frac{100\angle 0^\circ}{R} = -(10\angle -210^\circ + 10\angle -150^\circ)$$

$$\frac{10\angle 0^\circ}{R} = -\left(\frac{\sqrt{3}}{2} - \frac{\sqrt{3}}{2}\right)$$

$$\frac{10\angle 0^\circ}{R} = \sqrt{3}$$

$$R = \frac{10 \angle 0^\circ}{\sqrt{3}} = 5.744 \Omega$$

29. The voltage $v(t)$ across the terminals a and b as shown in the figure, is a sinusoidal voltage having a frequency $\omega = 100$ radian/s. When the inductor current $i(t)$ is in phase with the voltage $v(t)$, the magnitude of the impedance Z (in Ω) seen between the terminals a and b is _____ (up to 2 decimal places).

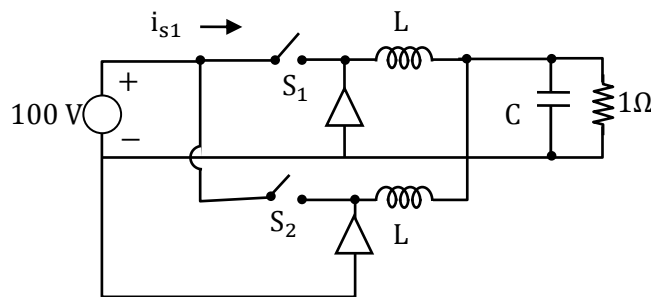


[Ans. *] Range: 50.0 to 50.0*

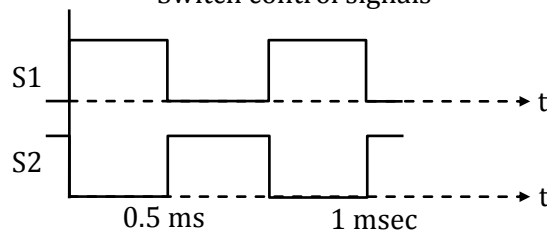
At resonance imaginary part of $Z_{eq} = 0$

$$\text{Real of } Z_{eq} = \frac{R_1 X_C^2}{R_1^2 + X_C^2} = \frac{100 \times 100 \times 100}{100^2 + 100^2} = 50 \Omega$$

30. The figure shows two buck converters connected in parallel. The common input dc voltage for the converters has a value of 100 V. The converters have inductors of identical value. The load resistance is 1 Ω . The capacitor voltage has negligible ripple. Both converters operate in the continuous conduction mode. The switching frequency is 1 kHz and the switch control signals are as shown. The circuit operates in the steady state. Assuming that the converters share the load equally. The average value of i_{s1} , the current of switch S1 (in Ampere), is (up to 2 decimal places).



Switch control signals



[Ans. *] Range: 11.50 to 13.50*

Hence it is a buck converter

$$V_0 = \alpha V_S$$

$$V_0 = 0.5 \times 100 = 50V$$

$$I_0 = \frac{V_0}{R} = \frac{50}{1} = 50A$$

$$V_S I_S = V_0 I_0$$

$$I_S = \frac{V_0 I_0}{V_S} = \frac{50 \times 50}{100} = 25 A$$

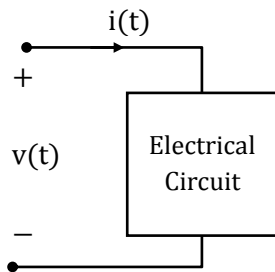
$$I_{S1} = I_{S2} = \frac{I_S}{2} = 12.5 A$$

31. The voltage across the circuit in the figure, and the current through it, are given by the following expressions:

$$v(t) = 5 - 10 \cos(\omega t + 60^\circ)V$$

$$i(t) = 5 + X \cos(\omega t)A$$

Where $\omega = 100 \pi$ radian/s. If the average power delivered to the circuit is zero, then the value of X (in Ampere) is _____ (up to 2 decimal places).



[Ans. *]Range: 10.0 to 10.0*

Given that, $v(t) = 5 - 10 \cos(\omega t + 60^\circ)$

$$i(t) = 5 + X \cos(\omega t - 0^\circ)$$

$$P_{\text{req}} = 0$$

$$0 = 5 \times 5 + \frac{1}{2} [(-10)(X) \cos(60^\circ)]$$

$$-25 = \frac{1}{2} [(-10)(X) \cos(60^\circ)]$$

$$X = 10$$

32. The per-unit power output of a salient-pole generator which is connected to an infinite bus is given by the expression. $P = 1.4 \sin \delta + 0.15 \sin 2\delta$. Where δ is the load angle. Newton-Raphson method is used to calculate the value of δ for $P = 0.8$ p.u. If the initial guess is 30° , then its value (in degree) at the end of the first iteration is

(A) 15°

(B) 28.48°

(C) 28.74°

(D) 31.20°

[Ans. C]

$$P(\delta) = 1.4 \sin \delta + 0.15 \sin 2\delta = 0.8$$

$$= 1.4 \sin \delta + 0.15 \sin 2\delta - 0.8 = 0$$

$$P'(\delta) = \frac{d}{d\delta} (P(\delta))$$

$$= 1.4 \cos \delta + 0.30 \cos 2\delta$$

Given $\delta_0 = 30^\circ$

By using Newton Raphson method for single variable

$$\Delta\delta = \delta_1 - \delta_0 = -\frac{f(\delta_0)}{f'(\delta_0)} = -\frac{P(\delta_0)}{P'(\delta_0)}$$

$$\delta_1 - 30^\circ = -\frac{1.4 \sin 30^\circ + 0.15 \sin 60^\circ - 0.8}{1.4 \cos 30^\circ + 0.3 \cos 60^\circ}$$

$$\delta_1 - 30^\circ = -0.0219 \text{ rad} = -1.26^\circ$$

$$\delta_1 = 28.74^\circ$$

33. Digital input signals A, B, C with A as the MSB and C as the LSB are used to realize the Boolean function $F = m_0 + m_2 + m_3 + m_5 + m_7$, where m_i denotes the i^{th} minterm. In addition, has a don't care for m_1 . The simplified expression for F is given by

- (A) $\bar{A}\bar{C} + \bar{B}C + AC$ (B) $\bar{A} + C$
(C) $\bar{C} + A$ (D) $\bar{A}C + BC + A\bar{C}$

[Ans. B*]

Given, $f = m_0 + m_2 + m_3 + m_5 + m_7$
and $m_1 = \text{dnt care}$

	$\bar{B}\bar{C}$	$\bar{B}C$	BC	$B\bar{C}$	
\bar{A}	1	X	1	1	$\bar{A} + C$
	0	1	3	2	
A		1	1		$f = \bar{A} + C$
	4	5	7	6	

34. Let $A = \begin{bmatrix} 1 & 0 & -1 \\ -1 & 2 & 0 \\ 0 & 0 & -2 \end{bmatrix}$ and $B = A^3 - A^2 - 4A + 5I$, Where I is the $I_{3 \times 3}$ identity matrix .The determinant of B is _____(Up to 1 decimal places)

[Ans. *]Range: 0.9 to 1.1

$$A = \begin{bmatrix} 1 & 0 & -1 \\ -1 & 2 & 0 \\ 0 & 0 & -2 \end{bmatrix}$$

$$|A - \lambda I| = 0$$

$$\begin{vmatrix} 1-\lambda & 0 & -1 \\ -1 & 2-\lambda & 0 \\ 0 & 0 & -2-\lambda \end{vmatrix} = 0$$

$$(1 - \lambda)((2 - \lambda)(-2 - \lambda)) - 1(0 - 0) = 0$$

$$\lambda = 1, 2, -2$$

Eigen values of A are 1, 2, -2

Eigen values of A^3 are 1, 8, -8

A^2 are 1, 4, 4

$4A$ are 4, 8, -8

$5I$ are 5, 5, 5

$A^3 - A^2 - 4A + 5I$ are 1, 1, 1

$$\therefore |B| = (1)(1)(1) = 1$$

35. A 3-phase 900 kVA, 3 kV / $\sqrt{3}$ kV (Δ/Y), 50 Hz transformer has primary (high voltage side) resistance per phase of 0.3Ω and secondary (low voltage side) resistance per phase of 0.02Ω . Iron loss of the transformer is 10 kW. The full load % efficiency of the transformer operated at unity power factor is _____ (up to 2 decimal places).

[Ans. *] Range: 97.20 to 97.55

Given $V_{L\Delta} = 3\text{kV}$

$V_{L\lambda} = \sqrt{3}\text{kV}$

Iron loss = 10 kW

$r_{\Delta}/\text{Phase} = 0.3 \Omega$

$r_{\lambda}/\text{Phase} = 0.02\Omega$

To find : η at full load and unity power factor

\therefore Rated phase current in Δ side = $\frac{900 \text{ kVA}}{3 \times V_{ph}} = \frac{900 \times 10}{3 \times 3 \times 10}$

$\Rightarrow I_{ph\Delta} = 100 \text{ A}$

\therefore Total copper loss in Δ side = $3(I_{ph\Delta}^2 r_{\Delta})$
 $= 3(100)^2 \times 0.3$
 $= 9\text{kW}$

And, Rated phase current in λ side = $\frac{900 \text{ kVA}}{\sqrt{3} \times V_R}$

$= 900 \times \frac{10^3}{\sqrt{3} \times \sqrt{3} \times 10^3}$

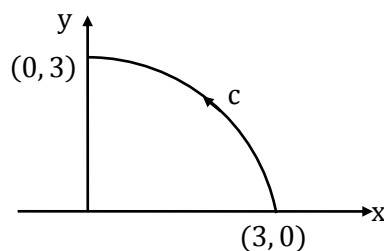
$\Rightarrow I_{ph\lambda} = 300\text{A}$

\therefore Total copper loss in λ side = $3(300)^2 r_{\lambda} = 5.4 \text{ Kw}$

\therefore Total copper loss P_{cetyl} at full load = 14.4 kW

$\therefore \eta = \frac{1 \times 900\text{K} \times 1}{900\text{K} + 14.4\text{K} + 10\text{K}}$
 $= \frac{900\text{K}}{924.4 \text{ K}} = 0.9736 \text{ or } 97.36\%$

36. As shown in the figure. C is the arc from the point (3,0) to the point (0,3) on the circle $x^2 + y^2 = 9$. The value of the integral $\int_C (y^2 + 2yx) dx + (2xy + x^2) dy$ is _____ (up to 2 decimal places).



[Ans. *] Range: 0.0 to 0.0*

$x^2 + y^2 = 9$

$x = 3 \cos \theta$

$$\begin{aligned}
 y &= 3 \sin \theta \\
 dx &= -3 \sin \theta d\theta \\
 dy &= 3 \cos \theta d\theta \\
 \theta &\text{ varies from } 0 \text{ to } \pi/2 \\
 \int (y^2 + 2xy) dx + (2xy + x^2) dy & \\
 &= \int_0^{\pi/2} (9 \sin^2 \theta + 18 \sin \theta \cos \theta) (-3 \sin \theta d\theta) + (18 \sin \theta \cos \theta + 9 \cos^2 \theta) (3 \cos \theta) d\theta \\
 &= \int_0^{\pi/2} (-27 \sin^3 \theta - 54 \sin^2 \theta \cos \theta + 54 \sin \theta \cos^2 \theta + 27 \cos^2 \theta) d\theta \\
 &= 0
 \end{aligned}$$

37. A 200 V DC series motor, when operating from rated voltage while driving a certain load, draws 10 A current and runs at 1000 r.p.m. The total series resistance is 1Ω . The magnetic circuit is assumed to be linear. At the same supply voltage, the load torque is increased by 44%. The speed of the motor in r.p.m. (rounded to the nearest integer) is _____

[Ans. *] Range: 823 to 827

From circuits diagram

$$E_{b1} = V - I_s(R_a + R_{se})$$

$$E_{b1} = 200 - 10(1) = 190V$$

$$N_1 = 1000 \text{ rpm}$$

Load torque increased by 44% ($T \propto I_a^2$)

$$\therefore \frac{T_2}{T_1} = \left(\frac{I_{a2}}{I_{a1}}\right)^2$$

$$\Rightarrow \frac{1.44T_1}{T_1} = \left(\frac{I_{a2}}{10}\right)^2$$

$$\therefore I_{a2}^2 = 144 \Rightarrow I_{a2} = 12 \text{ A}$$

At same voltage $E_{b2} = V - I_{a2}(R_a + R_{se})$

$$\therefore E_{b2} = 200 - 12(1) = 188V$$

For series motor . $N \propto \frac{E_b}{\phi}$ ($\phi \propto I_a$)

$$\frac{N_2}{N_1} = \frac{E_{B2}}{E_{b1}} \times \frac{I_{a1}}{I_{a2}}$$

$$\therefore \frac{N_2}{1000} = \frac{188}{190} \times \frac{10}{12}$$

$$N_2 = 824.56 \text{ rpm} \approx 825 \text{ rpm}$$

38. The positive, negative and zero sequence impedances of a three phase generator are Z_1 , Z_2 and Z_0 respectively. For a line-to-line fault with fault impedance Z_f , the fault current is $I_{f1} = kI_f$. Where I_f is the fault current with zero fault impedance. The relation between Z_f and k is _____

$$(A) Z_f = \frac{(Z_1 + Z_2)(1 - k)}{k}$$

$$(B) Z_f = \frac{(Z_1 + Z_2)(1 + k)}{k}$$

$$(C) Z_f = \frac{(Z_1 + Z_2)k}{1 - k}$$

$$(D) Z_f = \frac{(Z_1 + Z_2)k}{1 + k}$$

[Ans. A]

Line to line fault

We know that for line to line fault

$$I_{f_1} = -\sqrt{3}j \frac{E_a}{Z_1 + Z_2 + Z_f} \dots \textcircled{1}$$

$$I_f = -\sqrt{3}j \frac{E_a}{Z_1 + Z_2 + 0} \dots \textcircled{2}$$

Given $I_{f_1} = kI_f$

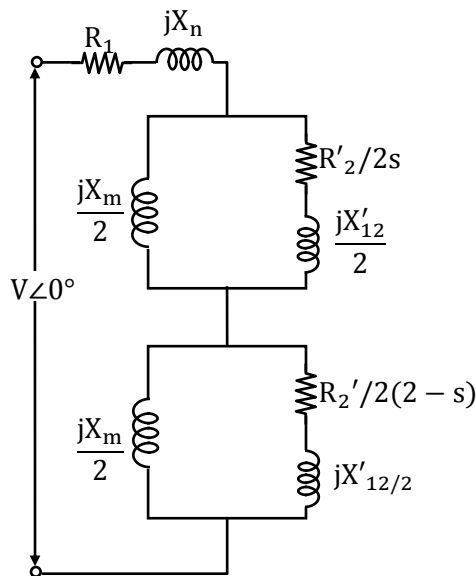
$$\frac{-\sqrt{3}IE_a}{Z_1 + Z_2 + Z_f} = k \times \frac{-\sqrt{3}IE_a}{Z_1 + Z_2}$$

$$Z_1 + Z_2 + Z_f = \frac{Z_1 + Z_2}{k}$$

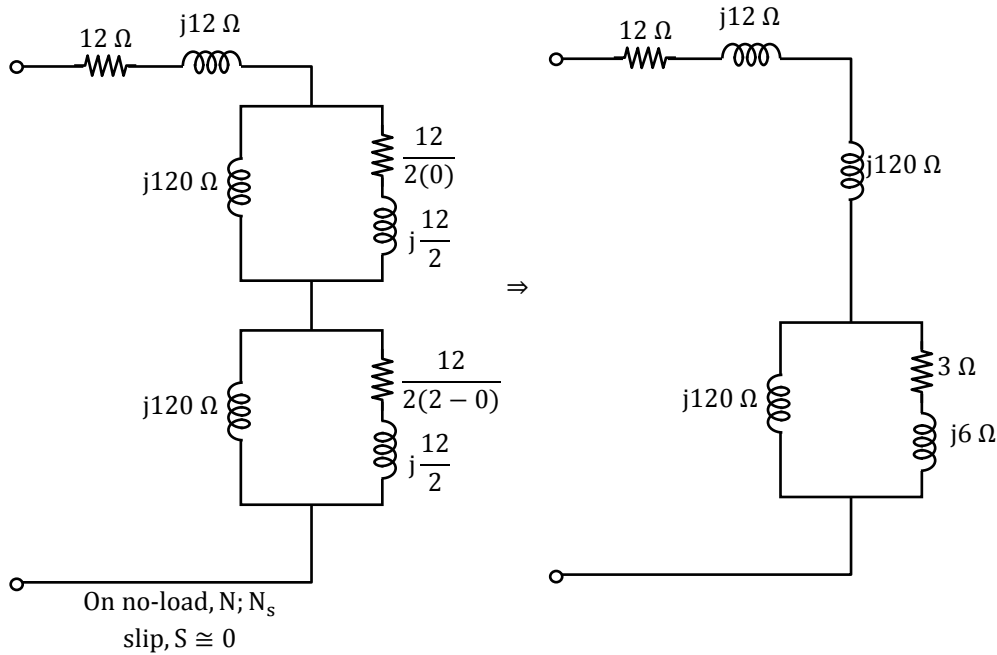
$$Z_f = \frac{Z_1 + Z_2}{k} - (Z_1 + Z_2)$$

$$Z_f = \frac{(1 - k)(Z_1 + Z_2)}{k}$$

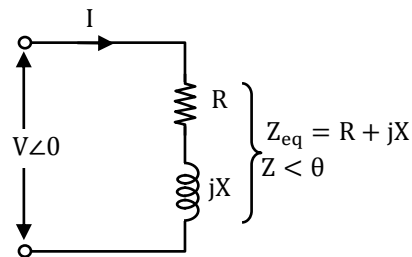
39. The equivalent circuit of a single phase induction motor is shown in the figure. Where the parameters are $R_1 = R_2' = X_{11} = X_{12}' = 12\Omega$, $X_M = 240\Omega$ and s is the slip. At no-load, the motor speed can be approximated to be the synchronous speed. The no-load lagging power factor of the motor is (up to 3 decimal places).



[Ans. *]Range: 0.104 to 0.112*



Simplifying the above circuit into a simple R-L circuit



$$Z_{eq} = \frac{(3 + j6)(j120)}{(3 + j126)} + [12 + j132]$$

Current drawn by motor is

$$I = \frac{V \angle 0}{Z \angle \theta} \{ \because \theta: \text{Impedance angle will be p. f. angle} \}$$

\therefore No-load lagging p.f. of motor is $(\cos \theta)$
 $\cos(83.9) = 0.106$ lagging power factor

40. Consider the two continuous time signals defined below.

$$x_1(t) = |t|, -1 \leq t \leq 1$$

$$= 0, \text{ Other wise}$$

$$x_2(t) = 1 - |t|, -1 \leq t \leq 1$$

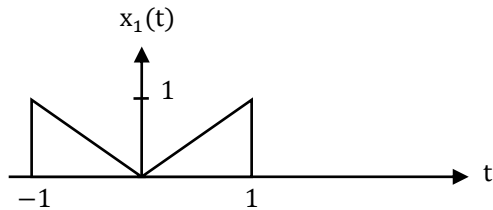
$$= 0, \text{ Other wise}$$

These signals are sampled with a sampling period of $T=0.25$ seconds to obtain discrete time signals $x_1(n)$ and $x_2(n)$, respectively. Which one of the following statements is true?

- (A) The energy of $x_1(n)$ is greater than the energy of $x_2(n)$.
- (B) The energy of $x_2(n)$ is greater than the energy of $x_1(n)$.
- (C) $x_1(n)$ and $x_2(n)$ have equal energies
- (D) Neither $x_1(n)$ nor $x_2(n)$ is a finite-energy signal

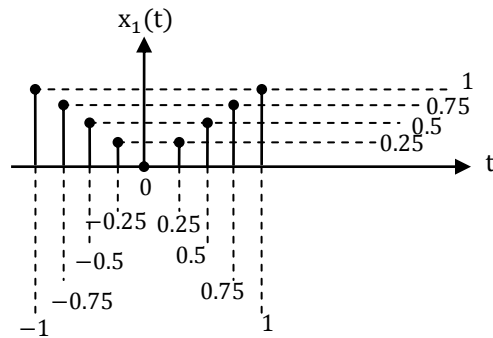
[Ans. A*]

$$x_1(t) = \begin{cases} |t|, & -1 \leq t \leq 1 \\ 0, & \text{otherwise} \end{cases}$$

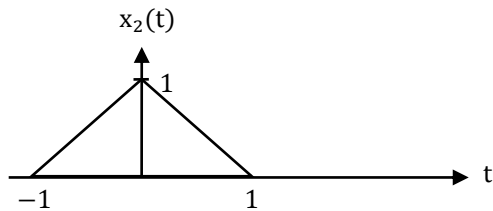


$T_s =$ Sampling time -period
 $= 0.25$ sec

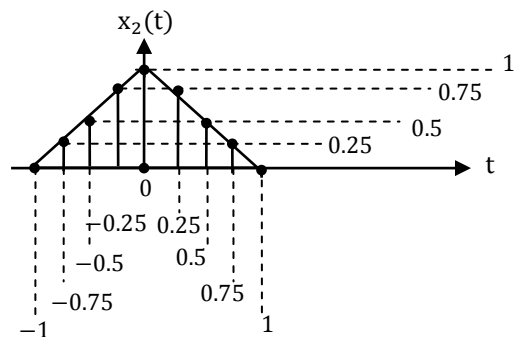
$$x_1(n) = \{1, 0.75, 0.5, 0.25, 0, 0.25, 0.5, 0.75, 1\}$$



Now, $x_2(t) = \begin{cases} 1 - |t|, & -1 \leq t \leq 1 \\ 0, & \text{otherwise} \end{cases}$



$$x_2(n) = \{0, 0.25, 0.5, 0.75, 1, 0.75, 0.5, 0.25, 0\}$$



Since $x_1(n)$ is having one more non-zero sample of amplitude '1' as compared to $x_2(n)$
Therefore, energy of $x_1(n)$ is greater than energy of $x_2(n)$

41. The signal energy of the continuous time signal

$$x(t) = [(t - 1)u(t - 1)] - [(t - 2)u(t - 2)] - [(t - 3)u(t - 3)] + [(t - 4)u(t - 4)]$$

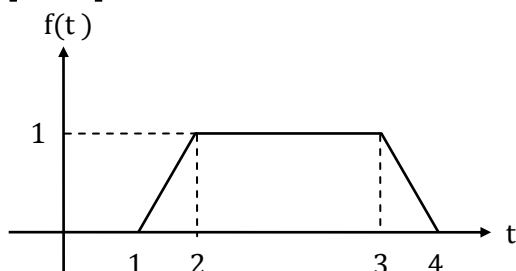
(A) 11/13

(B) 7/3

(C) 8/3

(D) 5/3

[Ans. D]



$$x(t) = [(t-1)u(t-1)] - [(t-2)u(t-2)] - [(t-3)u(t-3)] + [(t-4)u(t-4)]$$

$$x(t) = r(t-1) - r(t-2) - r(t-3) + r(t-4)$$

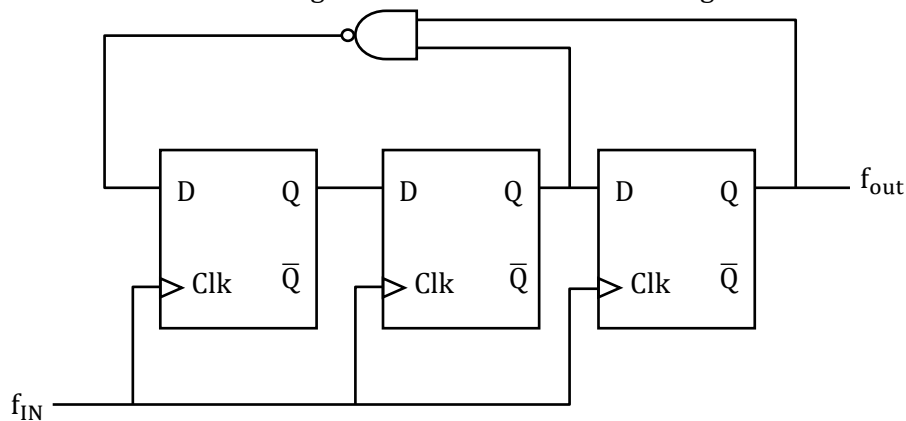
$$E = \int_1^4 |f(t)|^2 dt$$

$$\Rightarrow \int_1^2 (t-1)^2 dt + \int_2^3 (1)^2 dt + \int_3^4 (-t+4)^2 dt$$

$$\Rightarrow \int_1^2 (t^2 - 2t + 1) dt + \int_2^3 (1) dt + \int_3^4 (t^2 - 8t + 16) dt$$

$$\Rightarrow \left(\frac{t^3}{3} - t^2 + t \right)_1^2 + 1 \left(\frac{t^3}{3} - \frac{8t^2}{2} + 16t \right)_3^4 = \frac{5}{3}$$

42. What one of the following statement is true about the digital circuit shown in the figure?



(A) It can be used for dividing the input frequency by 3

(B) It can be used for dividing the input frequency by 5

(C) It can be used for dividing the input frequency by 7

(D) It cannot be reliably used as a frequency divider due to disjoint internal cycles

[Ans. B]

	Q _B Q _C D _A	Q _A D _B	Q _B D _C	Q _A	Q _B	Q _C	
	Initial			0	0	0	
1	1	0	0	1	0	0	} → 5 states
2	1	1	0	1	1	0	
3	1	1	1	1	1	1	
4	0	1	1	0	1	1	
5	0	0	1	0	0	1	
6	1	0	0	1	0	0	

MOD 5 counter → $f_{out} = \frac{f_m}{5}$

43. If C is a circle $|Z|=4, f(z) = \frac{z^2}{(z^2-3z+2)^2}$, then $\oint_C f(z) dz$ is _____

- (A) 1 (B) 0
(C) -1 (D) -2

[Ans. B*]

$$\int \frac{z^2}{(z^{-3}z + 2)} dz$$

$$\int \frac{z^2}{(z-1)^2(z-2)^2} dz$$

$$\text{Res. } f(z) = \lim_{z \rightarrow 1} \frac{1}{1!} \frac{d}{dz} \left((z-1)^2 \cdot \frac{z^2}{(z-1)^2(z-2)^2} \right)$$

$$= \lim_{z \rightarrow 1} \left(\frac{2z(z-2)^2 - 2z^2(z-2)}{(z-2)^2} \right)$$

$$= \lim_{z \rightarrow 1} \left(\frac{2z(z-2) - 2z^2}{(z-2)^2} \right) = \frac{-4}{-1} = 4$$

$$\text{Res. } f(z) = \lim_{z \rightarrow 2} \frac{1}{1!} \frac{d}{dz} \left((z-2)^2 \cdot \frac{z^2}{(z-1)^2(z-2)^2} \right)$$

$$= \lim_{z \rightarrow 2} \left(\frac{(z-1)^2 \cdot 2z - z^2 \cdot 2(z-2)}{(z-1)^4} \right)$$

$$= \lim_{z \rightarrow 2} \left(\frac{2z(z-1) - 2z^2}{(z-1)^3} \right) = \frac{4-8}{1} = -4$$

By residue theorem, $I = 2\pi i(4 - 4) = 0$

44. Let $f(x) = 3x^3 - 7x^2 + 5x + 6$. The maximum value of $f(x)$ over the interval $[0, 2]$ is _____ (up to two decimal places)

[Ans. *] Range: 11.5 to 12.5*

$$f(x) = 3x^3 - 7x^2 + 5x + 6 \quad \text{in } [0, 2]$$

$$f'(x) = 9x^2 - 14x + 5$$

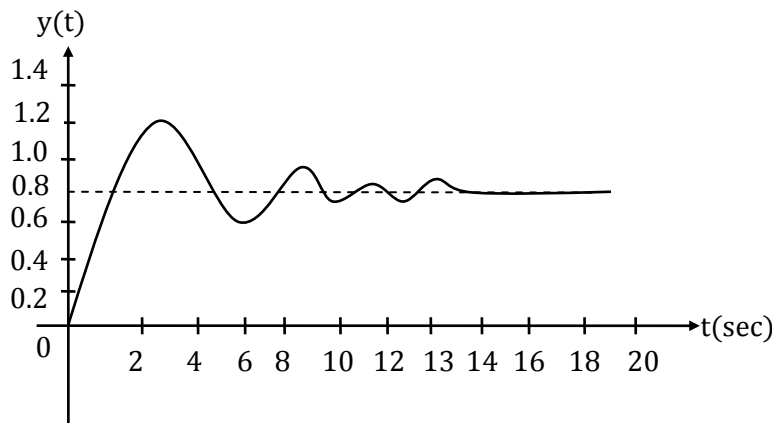
$$f''(x) = 18x - 14$$

$$f'(x) = 0$$

$$x^2 - 14x + 5 = 0$$

$x = 1, 0.55$
 $x = 1$
 $f''(1) = 18 - 14 = 4 > 0$ minima
 $x = 0.55$
 $f''(0.55) = -4.1 < 0$ minima
 Maximum $\{f(0), f(0.55), f(2)\}$
 Maximum $\{6, 7.13, 12\} = 12$

45. The unit step response $y(t)$ of unity feedback system with open loop transfer function $G(s)H(s) = \frac{k}{(s+1)^2(s+2)}$ is shown in the figure. The value of k is _____ (up to two decimal places)



[Ans. *]Range: 8.0 to 8.0

$$C(s)/R(s) = \frac{\frac{k}{(s+1)^2(s+2)}}{1 + \frac{k}{(s+1)^2(s+2)} \cdot 1}$$

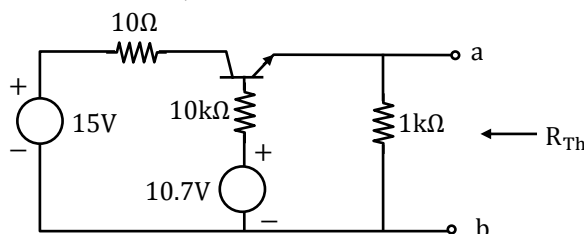
$$\frac{C(s)}{R(s)} = \frac{k}{(s+1)^2(s+2) + k}$$

$$C(s) = \lim_{s \rightarrow 0} s \cdot \frac{1}{s} \left[\frac{k}{(s+1)^2(s+2) + k} \right]$$

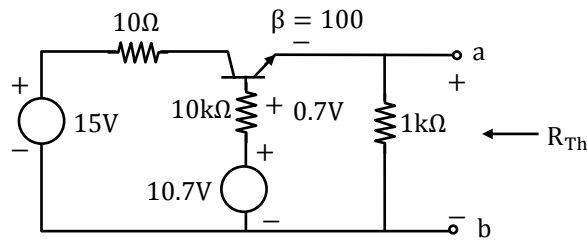
$$0.8 = \frac{k}{k+2} \Rightarrow k = 0.8k + 1.6$$

$$0.2k = 1.6; k = 8$$

46. In the circuit shown in the figure. The bipolar junction transistor (BJT) has a current gain $\beta = 100$. The base-emitter voltage drop is a constant. $V_{BE} = 0.7$ V. The value of the Thevenin's equivalent resistance R_{th} (in Ω) as shown in the figure is _____ (up to 2 decimal places).



[Ans. *] Range: 89.00 to 91.00



$V_{th} = V_{ab} =$ Voltage across $1k\Omega$

Taking KVL in emitter loop,

$$10.7 - 0.7 - (10k)i_b - (1k)(\beta + 1)i_b = 0$$

$$10 = 10k i_b + 101ki_b = 111ki_b$$

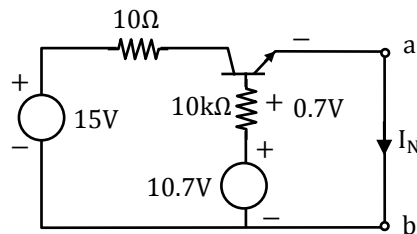
$$i_b = \frac{10}{111k}$$

$$\therefore i_e = \frac{10}{111k} \times 101 = 10 \times \frac{101}{111} \times 10^{-3} = 9.099 \text{ mA}$$

$$\therefore V_{th} = 1k \times 9.099m = 9.099V$$

Now, For I_N (Norton's Current), short ab $\therefore 1k\Omega$ resistance will be ineffective

Now, taking KVL,

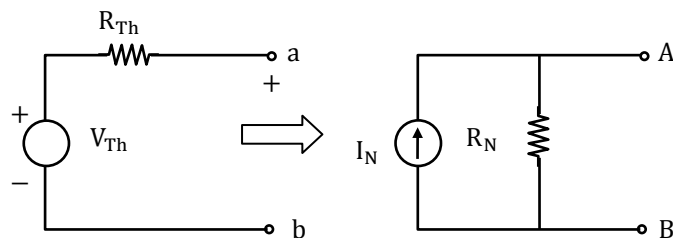


$$10.7 - 0.7 - 10ki_b = 0$$

$$i_b = \frac{10}{10k} = 1 \text{ mA}$$

Note:

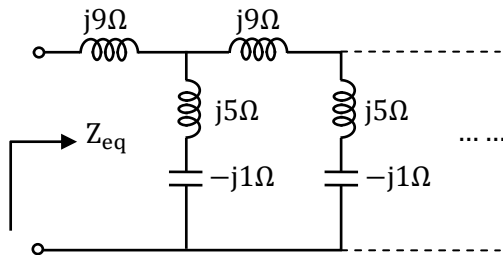
Now, for a same circuit Thevenin's resistance and Norton's resistance are same



$$\therefore R_{th} = \frac{V_{th}}{I_N} = \frac{9.099V}{101mA}$$

$$R_{th} \approx 90.09\Omega \approx 90.1\Omega$$

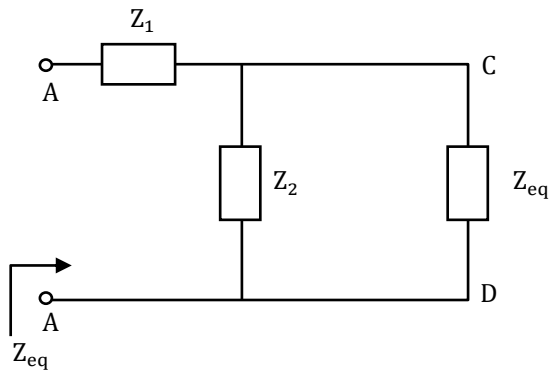
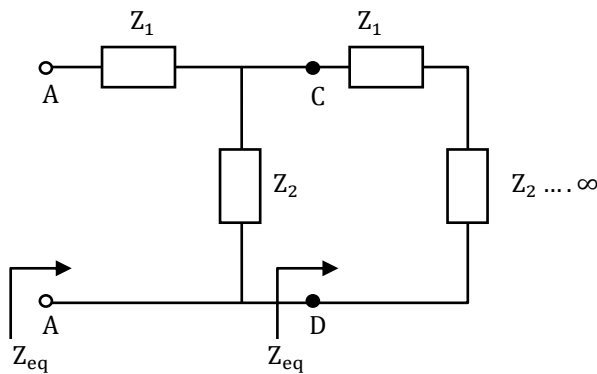
47. The equivalent impedance Z_{eq} for the infinite ladder circuit shown in the figure is



- (A) $j12\ \Omega$
(C) $j13\ \Omega$

- (B) $-j12\ \Omega$
(D) $13\ \Omega$

[Ans. A]



$$Z_1 = j9$$

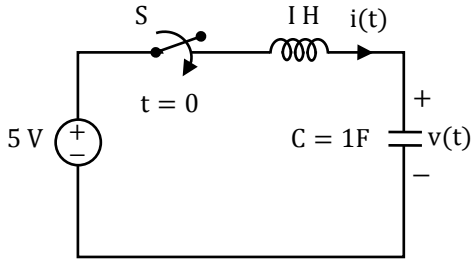
$$Z_2 = j5 - j1 = j4$$

$$Z_{eq} = Z_1 + \frac{Z_2 Z_{eq}}{Z_2 + Z_{eq}}$$

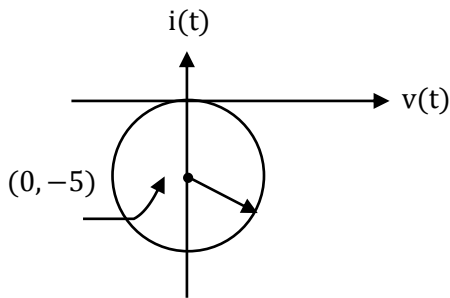
By solving above solution

$$Z_{eq} = j12$$

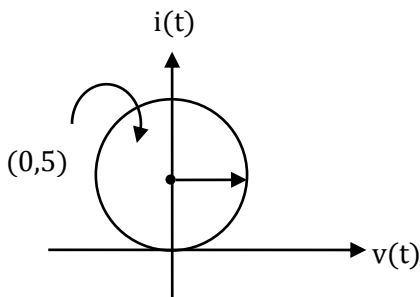
48. A DC voltage source is connected to a series L-C circuit by turning on the switch S at time $t = 0$ as shown in the figure. Assume $i(0) = 0, v(0) = 0$. Which one of the following circular loci represents the plot of $i(t)$ versus $v(t)$?



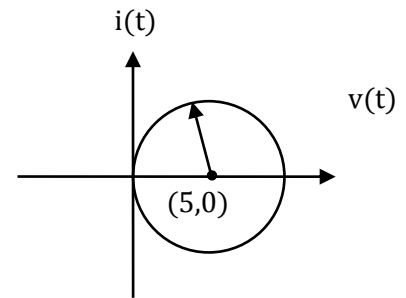
(A)



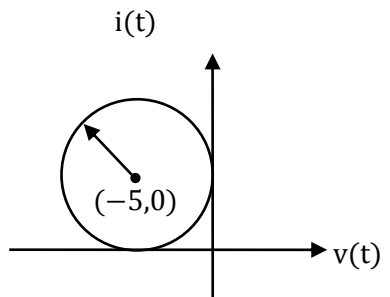
(C)



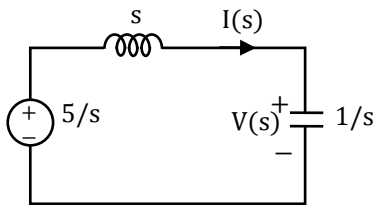
(B)



(D)



[Ans. B*]



$$I(s) = \frac{5/s}{s + \frac{1}{s}} = \frac{5}{s^2 + 1}$$

$$i(t) = 5 \sin t$$

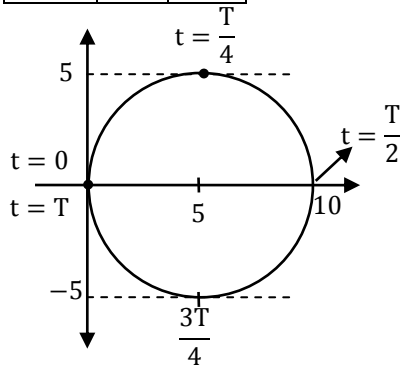
$$v(t) = \frac{1}{C} \int_0^t i dt = \int_0^t 5 \sin t dt$$

$$v(t) = 5[-\cos t]_0^t = 5[-\cos t + 1]$$

$$v(t) = 5 - 5 \cos t$$

t	i(t)	v(t)
0	0	0

$T/4$	5	5
$T/2$	0	10
$3T/4$	-5	5
T	0	0

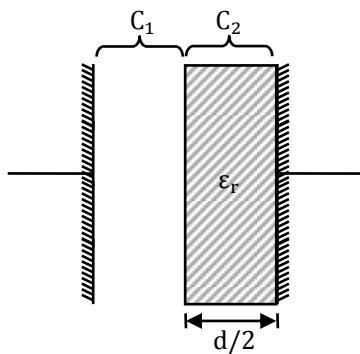


49. The capacitance of an air-filled parallel-plate capacitor is 60 pF. When a dielectric slab whose thickness is half the distance between the plates, is placed on one of the plates covering it entirely, the capacitance becomes 86 pF. Neglecting the fringing effects. The relative permittivity of the dielectric is _____ (up to 2 decimal places).

[Ans. *] Range: 2.50 to 2.55

Given: $C = \frac{\epsilon_0 A}{d} = 60 \text{ pF}$

In second case:



$$\begin{aligned} \text{Capacitance } C_1 &= \frac{\epsilon_0 A}{d/2} \\ &= \frac{2\epsilon_0 A}{d} = 2 \times (60 \text{ pF}) = 120 \text{ pF} \end{aligned}$$

$$\text{And } C_2 = \frac{2\epsilon_0 \epsilon_r A}{d} = (2 \times 30) \epsilon_r \text{ pF} = 120 \epsilon_r \text{ pF}$$

$$\text{Now, } C_{\text{eq}} = \frac{C_1 C_2}{C_1 + C_2} = \frac{120 \times 120 \epsilon_r}{(120 + 120 \epsilon_r)} \text{ pF} = 86 \text{ pF} \quad (\text{Given})$$

$$\text{or, } 86 = \frac{120 \times 120 \epsilon_r}{120(1 + \epsilon_r)}$$

$$\frac{86}{120} = \frac{\epsilon_r}{1 + \epsilon_r}$$

$$\text{or, } \epsilon_r = \frac{86}{34} = 2.53$$

50. A 0-1 Ampere moving iron ammeter has an internal resistance of 50 mΩ and inductance of 0.1 mH. A shunt coil is connected to extend its range to 0-10 Ampere for all operating frequencies. The time constant in milliseconds and resistance in mΩ of the shunt coil respectively are
- (A) 2, 5.55 (B) 2, 1
(C) 2.18, 0.55 (D) 11.1, 2

[Ans. A*]

Given, $I_m = 1A$, $R_m = 50 \text{ m}\Omega$
 $L_m = 0.1 \text{ mH}$, $I = 10 \text{ a}$

We know

$$R_{sh} = \frac{R_m}{(m - 1)}; m = \frac{I}{I_m}$$

Here, $\frac{10}{1} = 10$

$$\therefore R_{sh} = \frac{50 \times 10^{-3}}{10 - 1} = \frac{50 \times 10^{-3}}{9} = 5.56 \text{ m}\Omega$$

For all frequencies time constants of the shunt and meter arm should be equal

i. e., $\frac{\omega L_m}{R_m} = \frac{\omega L_{sh}}{R_{sh}}$

or, $\frac{L_m}{R_m} = \frac{L_{sh}}{R_{sh}}$

or, $\frac{L_m}{R_m} = \frac{0.1 \times 10^{-3}}{50 \times 10^{-3}} = 0.002 = 2 \text{ ms}$

\therefore Option (A) is correct

51. The Fourier transform of a continuous-time signal $x(t)$ is given by $X(j\omega) = \frac{1}{(10+j\omega)^2}$, $-\infty < \omega < \infty$, Where $j = \sqrt{-1}$ and ω denotes frequency. Then the value of $|\ln x(t)|$ at $t = 1$ is (up to 1 decimal place). (It denotes the logarithm to base e)

[Ans. *] Range: 9.5 to 10.5*

$$x(t) \Leftrightarrow X(j\omega) = \frac{1}{(10 + j\omega)^2}$$

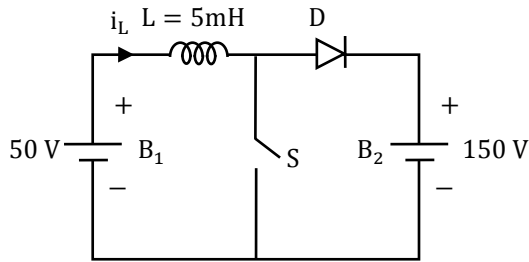
By taking inverse Fourier transform

$$x(t) = te^{-10t}u(t)$$

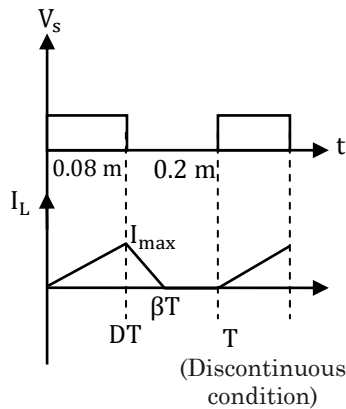
Now, $x(t)_{t=1} = 1 \times e^{-10} \times 1 = e^{-10}$

Thus, $|\ln\{x(t)\}| = |\ln(e^{-10})|$
 $= |-10| = 10$

52. A dc to dc converter shown in the figure is charging a battery bank. B2 whose voltage is constant at 150 V. B1 is another battery bank whose voltage is constant at 50 V. The value of the inductor. L is 5 mH and the ideal switch, S is operated with a switching frequency of 5 kHz with a duty ratio of 0.4. Once the circuit has attained steady state and assuming the diode D to be ideal. The power transferred from B1 to B2 (in Watt) is _____ (up to 2 decimal places).



[Ans.*] Range: 12 to 12



Given circuit is a boost converter where for continuous condition

$$V_0 = \frac{V_s}{(1 - D)} \quad \dots \textcircled{1}$$

Given data

$$V_s = V_{B1} = 50 \text{ V}, D = 0.4$$

$$L = 5 \text{ mH}$$

$$V_{B2} = 150 \text{ V}$$

Putting the values in $\textcircled{1}$ we get

$$\begin{aligned} V_0 &= \frac{50}{1 - 0.4} \\ &= \frac{50}{0.6} \\ &= \frac{500}{6} = 83.33 \text{ V} \end{aligned}$$

$$\text{Then } V_0 = \left(\frac{\beta}{\beta - D} \right) V_0$$

$$150 = \frac{\beta}{(\beta - 0.4)} \quad (50)$$

$$3\beta - 1.2 = \beta$$

$$2\beta = 1.2$$

$$\beta = 0.6$$

The inductor discharges during $0.4T$ to $0.6T$ and power transferred from V_s to V_0 .

The value of I_{\max} is given by

$$\therefore 50 = L \left(\frac{I_{\max} - 0}{DT - 0} \right)$$

$$= 50 \left(\frac{I_{\max}}{(0.4) \left(\frac{1}{5K} \right)} \right)$$

$$= \frac{(5m)I_{\max}}{0.08m}$$

$$I_{\max} = 0.8 \text{ A}$$

∴ Energy transferred is given by charging and discharging period of inductor.

$$= 150 \times \frac{1}{2} \times I_{\max}(0.6T - 0.4T)$$

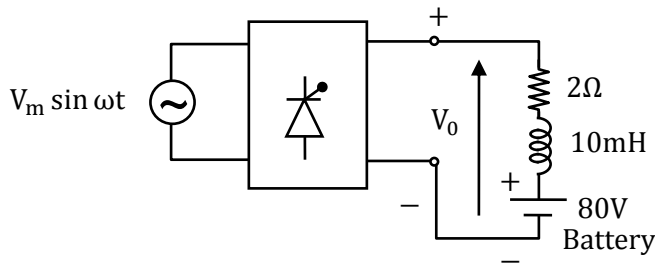
$$= 150 \times \frac{1}{2} \times 0.8(0.2T)$$

$$= 15 \times 0.8T$$

$$= 12 \left(\frac{1}{5K} \right)$$

$$\text{Power} = \frac{(12) \left(\frac{1}{5K} \right)}{\left(\frac{1}{5K} \right)} = 12 \text{ W}$$

53. A phase controlled single phase rectifier, supplied by an AC source, feeds power to an R-L-E load as shown in the figure. The rectifier output voltage has an average value given by $V_0 = \frac{V_m}{2\pi} (3 + \cos \alpha)$, where $V_m = 80\pi$ volts and α is the firing angle. If the power delivered to the lossless battery is 1600 W. α in degree is _____ (up to 2 decimal places).



[Ans. 90] Range: 90.0 to 90.0*

$$V_0 = \frac{V_m}{2\pi} (3 + \cos \alpha)$$

$$E_n I_0 = 1600 \text{ W}$$

$$I_0 = \frac{1600}{80} = 20 \text{ A}$$

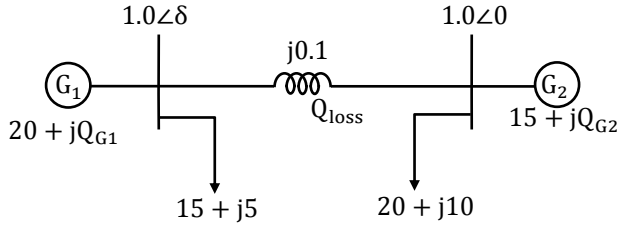
$$V_0 = E_b + I_0 R_a$$

$$\frac{V_m}{2\pi} (3 + \cos \alpha) = 80 + (20 \times 2)$$

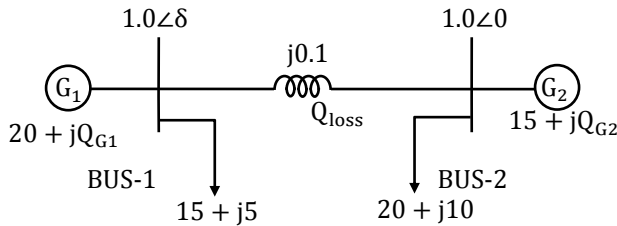
$$\frac{80\pi}{2\pi} (3 + \cos \alpha) = 80 + 40$$

$$\alpha = 90^\circ$$

54. Consider the two bus power system network with given loads as shown in the figure. All the values shown in the figure are in per unit. The reactive power supplied by generator G_1 and G_2 are Q_{G1} and Q_{G2} respectively. The per unit values of Q_{G1} , Q_{G2} , and line reactive power loss (Q_{loss}) respectively are
- (A) 5.0,12.68,2.68 (B) 6.34,10.00,1.34
(C) 6.34,11.34,2.68 (D) 5.00,11.34,1.34



[Ans. C]



Real power generated by $G_1 = 20$

and load at Bus 1 = 15

So, real power flow from Bus 1 to Bus 2 = $20 - 15 = 5$

So $P = \frac{V_1 V_2}{X} \sin \delta$ (We know)

$$5 = \frac{1 \times 1}{0.1} \sin(\delta - 0)$$

$$\sin \delta = 5 \times 0.1 = \frac{1}{2}$$

$$\delta = 30^\circ$$

$$Q_{Line} = I^2 X_L$$

$$= \left(\frac{1 \angle 30^\circ - 1 \angle 0^\circ}{0.1 \angle 90^\circ} \right)^2 \times 0.1 \angle 90^\circ$$

$$= 2.68 \text{ J}$$

$$Q_s = \frac{V_s^2}{X_L} = \frac{V_s V_R}{X_L} = \cos \delta$$

$$= \frac{1 \times 1}{0.1} - \frac{1 \times 1}{0.1} \cos 30^\circ$$

$$= 10 - 5\sqrt{3} = 1.34 \text{ J}$$

$$Q_R = \frac{V_s \cdot V_R}{X_L} \cos \delta - \frac{V_s^2}{X_L}$$

$$= \frac{1 \times 1}{0.1} - \cos 30^\circ - \frac{1^2}{0.1}$$

$$= 5\sqrt{3} - 10 = 1.34 \text{ J}$$

$$Q_{G1} = 5 + 1.34 = 6.345$$

$$Q_{g2} = 10 + 1.35 = 11.34 \text{ J}; \quad Q_{loss} = 1.34 + 1.34 = 2.68$$

55. The number of roots of the polynomial. $s^7 + s^6 + 7s^5 + 14s^4 + 31s^3 + 73s^2 + 25s + 200$, in the open left half of the complex plane is

- (A) 3 (B) 4
(C) 5 (D) 6

[Ans .A]

s^7	1	7	31	25	
s^6	1	14	73	200	
s^5	-7	-42	-175	0	
$A(s) * s^4$	+8	+48	200		$A(s) = 8s^4 + 48s^2 + 200$
s^3	0(32)	0(96)	0		$\frac{dA(s)}{ds} = 32s^3 + 96s$
s^2	24	200			
s^1	-170.67				
s^0	200				

Number of sign changes in 1st column shows that
Roots lie in RHS

