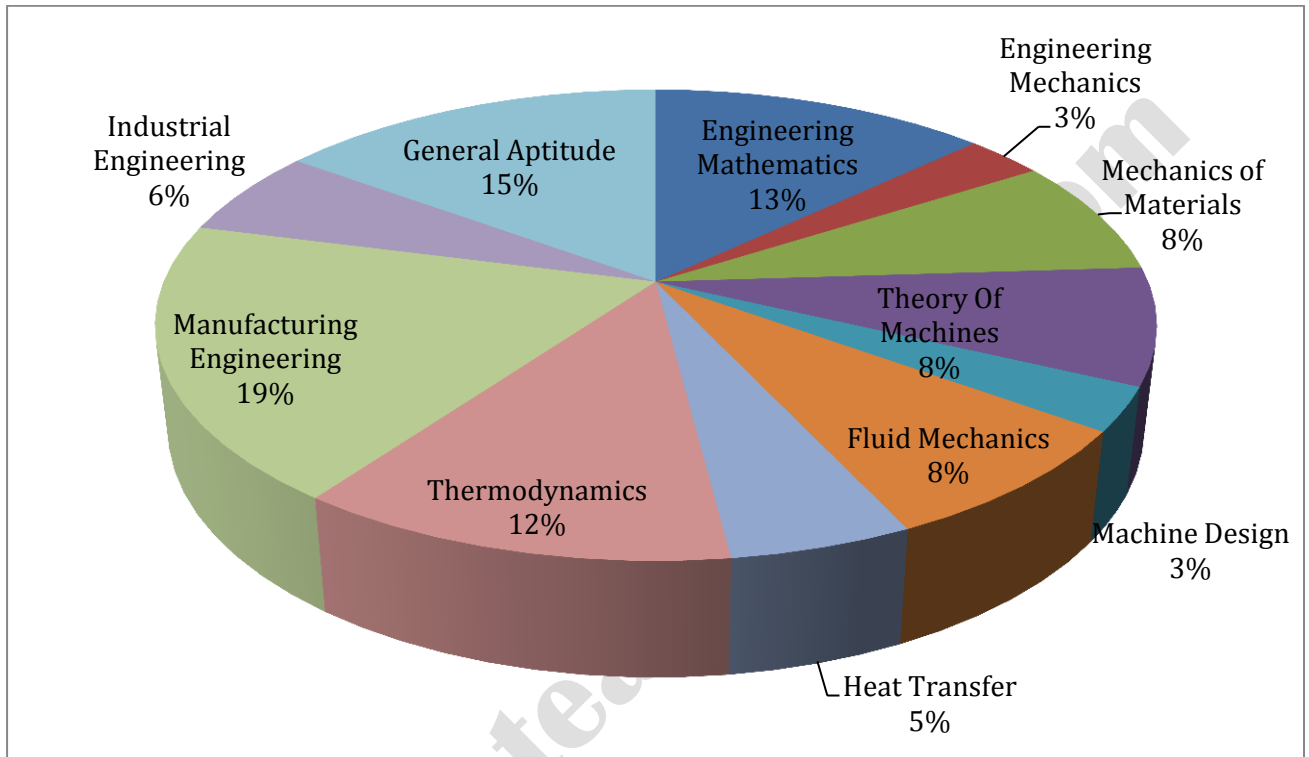


## ANALYSIS OF GATE 2018

### Mechanical Engineering



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### ME ANALYSIS-2018\_3-Feb\_Afternoon

SUBJECT	No. of Ques.	Topics Asked in Paper(Memory Based)	Level of Ques.	Total Marks
Engineering Mathematics	1 Marks: 5 2 Marks: 4	Random Variable, Complex Variable, Divergence, Complementary Function, Determinant, Variable Separable Method, Fourier Series	Easy	13
Engineering Mechanics	1 Marks:1 2 Marks: 1	Slider Crank Mechanism, Collision	Medium	3
Mechanics of Materials	1 Marks: 2 2 Marks: 3	Columns, Simple Stress and Strain	Medium	8
Theory Of Machines	1 Marks: 2 2 Marks: 3	Gear Strain, Vibration, Torsion, Cams	Medium	8
Machine Design	1 Marks: 1 2 Marks: 2	Bearing Capacity, Breaks	Easy	3
Fluid Mechanics	1 Marks: 2 2 Marks: 3	Fluid Properties, Flow through pipes	Medium	8
Heat Transfer	1 Marks: 1 2 Marks: 2	Radiation, Convection	Easy	5
Thermodynamics	1 Marks: 2 2 Marks: 5	Ideal Gas, IC Engine, Vapour compression cycle, Refrigeration	Medium	12
Manufacturing Engineering	1 Marks: 7 2 Marks: 6	Milling, Metal Cutting, Forming, EDM	Tough	19
Industrial Engineering	1 Marks: 2 2 Marks: 2	Inventory Management, Linear Programming	Tough	6
General Aptitude	1 Marks: 5 2 Marks: 5	Geometry, TSD, Functions, Grammar, Numbers, Work, inference	Easy	15
<b>Total</b>	<b>65</b>			<b>100</b>
<b>Faculty Feedback</b>	Majority of the question were concept based. General Aptitude And Mathematics is Very Easy. Core Subject Questions were 50% easy, 30% medium and 20% tough.			

## GATE 2018 Examination Mechanical Engineering

Test Date: 3-Feb-2018

Test Time: 2:00 PM 5:00 PM

Subject Name: Mechanical Engineering

### General Aptitude

**Q.1 - Q.5 Carry One Mark each.**

- "The dress \_\_\_ her so well that they all immediately \_\_\_ her on her appearance."  
The words that best fill the blanks in the above sentence are

(A) Complemented, Complemented	(B) Complimented, Complemented
(C) Complimented, Complimented	(D) Complemented, Complimented

**[Ans. D]\***  
Complemented-Complete  
Complimented -Remark of admiration  
Complement -To contribute extra features to something so as to improve or emphasize their qualities  
Compliment-Politely congratulate or praise someone /something
- Find the missing group of letters in the following series:  
BC, FGH, LMNO, \_\_\_

(A) UVWXY	(B) TUVWX
(C) STUVW	(D) RSTUV

**[Ans. B]\***  
BC, FGH, LMNO, TUVWX
- The perimeters of a circle, a square and an equilateral triangle are equal. Which one of the following statements is true?

(A) The circle has the largest area  
(B) The square has the largest area.  
(C) The equilateral triangle has the largest area.  
(D) All the three shapes have the same area.

**[Ans. A]\***

$$P_c = P_s = P_t$$

$$\pi D = 4a = 3s$$

$$\text{now, Area of circle} = \frac{\pi}{4} D^2 \quad [\pi D = 4a]$$

$$\text{Area of square} = a^2$$

$$= \frac{\pi^2 D^2}{16} = \frac{\pi^2}{16} D^2$$

$$\text{Area of triangle} = \frac{\sqrt{3}}{4} s^2 = \frac{\sqrt{3}}{4} \left(\frac{\pi D}{3}\right)^2 \quad \{\because \pi D = 3s\}$$

$$= \frac{(\sqrt{3})\pi^2 D^2}{36}$$

$$\frac{\pi}{4} > \frac{\pi^2}{16} > \frac{\sqrt{3}\pi^2}{36}$$

So, circle has the largest area

4. The value of the expression  $\frac{1}{1+\log_u vw} + \frac{1}{1+\log_v wu} + \frac{1}{1+\log_w uv} = ?$
- (A) -1 (B) 0  
(C) 1 (D) 3

[Ans. C]

$$\begin{aligned} & \frac{1}{1+\log_u vw} + \frac{1}{1+\log_v wu} + \frac{1}{1+\log_w uv} \\ &= \frac{1}{\log_u u + \log_u vw} + \frac{1}{\log_v v + \log_v wu} + \frac{1}{\log_w w + \log_w uv} \\ &= \frac{1}{\log_u uvw} + \frac{1}{\log_v uvw} + \frac{1}{\log_w uvw} \\ &= \log_{uvw} u + \log_{uvw} v + \log_{uvw} w = \log_{uvw} uvw = 1 \end{aligned}$$

5. "The judge's standing in the legal community though shaken by false allegations of wrong doing, remained \_\_\_\_"
- The word that best fills the blank in the above sentence is
- (A) Undiminished (B) damaged  
(C) illegal (D) Uncertain

[Ans. A]

Undiminished -unambitious /unskilled

Undiminished means not diminished, reduced or lessened. It is integrated, through and complete in nature

**Q.6 - Q.10 Carry Two Mark each.**

6. A wire would enclose an area of 1936 m<sup>2</sup>, if it is bent into a square. The wire is cut into two pieces. The longer piece is thrice as long as the shorter piece. The long and the short pieces are bent into a square and a circle, respectively. Which of the following choices is closes to the sum of the areas enclosed by the two pieces in square meters.
- (A) 1096 (B) 1111  
(C) 1243 (D) 2486

[Ans. C]

$$\begin{aligned} \text{Initial square area} &= 1936 \text{ m}^2 \\ a^2 &= 1936 \text{ m}^2 \\ a &= 44\text{m} \end{aligned}$$

$$\begin{aligned} \text{Length of wire} &= 4a \\ &= 4 \times 44 = 176 \text{ m} \end{aligned}$$

$$\begin{aligned} \text{Now, part - 1 length} &= 3 \times 44 = 132 \text{ m} \\ \text{part - 2 length} &= 1 \times 44 = 44\text{m} \end{aligned}$$

Long wire is bent in square,

$$\text{So, } 4a = 132$$

$$a = 33\text{m}$$

$$\text{Area of square} = 33^2 = 1089 \text{ m}^2$$

Now, small wire is bent in circle

$$\text{So, circle perimeter, } \pi D = 44$$

$$\frac{22}{7} \times D = 44$$

$$D = 14 \text{ m}$$

$$\begin{aligned} \text{Area of circle} &= \frac{\pi}{4} \times D^2 = \frac{\pi}{4} \times 14^2 \\ &= 153.94 \text{ m}^2 \end{aligned}$$

$$\begin{aligned} \text{Total area enclosed} &= \text{Area of square} + \text{Area of circle} \\ &= 1089 + 153.94 \\ &= 1242.97 \approx 1243 \text{ m}^2 \end{aligned}$$

7. A house has a number which needs to be identified. The following three statements are given that can help in identifying the house member.

- i. If the house number is a multiple of 3m then it is number from 50 to 59.
- ii. If the house number NOT a multiple of 4, then it is a number from 60 to 69.
- iii. If the house number is NOT a multiple of 6, then it is a number from 70 to 79.

What is the house number?

- (A) 54 (B) 65  
(C) 66 (D) 76

**[Ans. D]**

By condition 1  $\Rightarrow$

Possible numbers are  $\rightarrow 51, 54, 57$

By condition 2  $\Rightarrow$

Possible numbers are  $\rightarrow 61, 62, 63, 65, 66, 67, 69$

By condition 3  $\Rightarrow$

Possible numbers are  $\rightarrow 70, 71, 73, 74, 75, 76, 77, 79$

$\Rightarrow 66$  is multiple of 3 and it does not belong to 50 to 59. So it will not be the answer.

$\Rightarrow 54$  is multiple of 3 and 6. But it is not the multiple of 4.

So according to second condition it cannot be the answer.

$\Rightarrow$  Because 65 is not the multiple of 6. So condition 3 is not satisfied. So it can not be the answer

$\Rightarrow$  For 76. All three conditions are satisfied. So it is the answer.

8. An unbiased coin is tossed six times in a row and four different such trials are conducted. One trial implies six tosses of the coin. If H stands for head and T stands for tail, the following are the observations from the four trials.

- (1) HTHTHT
- (2) TTHHHT
- (3) HTTHHT
- (4) HHHT\_\_

Which statement describing the last two coin tosses of the fourth trial has the highest probability of being correct?

- (A) Two T will occur
- (B) One H and one T will occur
- (C) Two H will occur
- (D) One H will be followed by one T.

**[Ans. B]\***

For remaining last two tosses possible cases are,

HH

HT

TH

TT

Out of 4 possible cases one head and T will have the highest probability of occurrence.

9. A contract is to be completed in 52 days and 125 identical robots were employed each operational for 7 hours a day. After 39 days, five-seventh of the work was completed. How many additional robots would be required to complete the work on time, if each robot is now operational for 8 hours a day?

- (A) 50 (B) 89  
(C) 146 (D) 175

**[Ans. \*] (This question is to be challenged)**

$$M=125$$

$$D=52$$

$$H=7$$

$$\frac{M_1 D_1 H_1}{W_1} = \frac{M_2 D_2 H_2}{W_2}$$

$$\text{Completed} = \frac{5}{7}$$

$$\frac{125 \times 39 \times 7}{\frac{5}{7}} = \frac{x \times 13 \times 8}{\frac{2}{7}} \quad \text{Remaining} = 1 - \frac{5}{7} = \frac{2}{7}$$

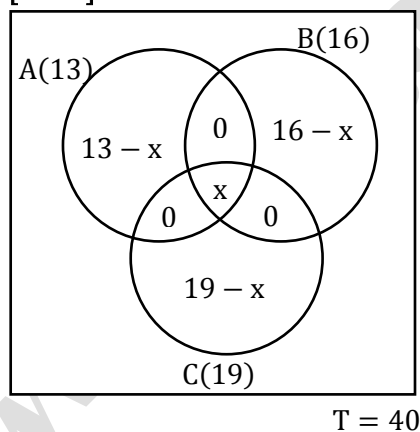
$$x \approx 131.01$$

$$\text{Additional robot required} = 131.25 - 125 = 6.25 \approx 7$$

10. Forty students watched films A, B and C over a week. Each student watched either only one film or all three. Thirteen students watched film A, sixteen students watched film B and nineteen students watched film C. How many students watched all three films?

- (A) 0 (B) 2  
(C) 4 (D) 8

**[Ans. C]**



## Technical

Q.1 - Q.25 Carry One Mark each.

1. If  $A = \begin{bmatrix} 1 & 2 & 3 \\ 0 & 4 & 5 \\ 0 & 0 & 1 \end{bmatrix}$  then  $\det(A^{-1})$  is \_\_\_\_\_ (correct to two decimal places)

[Ans. \*] Range: 0.25 to 0.25

$$|A| = 1 \times 4 \times 1 = 4$$

$$|A^{-1}| = \frac{1}{|A|} = 1/4 = 0.25$$

2. A hollow circular shaft of inner radius 10 mm, outer radius 20 mm and length 1 m is to be used as a torsional spring. If the shear modulus of the material of the shaft is 150GPa, the torsional stiffness of the shaft (in kN-m/rad) is\_\_\_\_ (correct to two decimal places).

[Ans.\*]Range: 35.24 to 35.44

$$R_i = 10 \text{ mm} \Rightarrow d_i = 20 \text{ mm}$$

$$R_o = 20 \text{ mm} \Rightarrow d_o = 40 \text{ mm}$$

$$L = 1 \text{ m}$$

$$G = 150 \text{ GPa}$$

$$\text{torsional stiffness} = k = \left(\frac{T}{\theta}\right) = \frac{GJ}{L}$$

$$= \frac{150 \times 10^9 \times \frac{\pi}{32} [(0.04)^4 - (0.02)^4]}{(1)}$$

$$= 35342.91 \text{ Nm/rad}$$

$$= 35.342 \text{ kNm/rad}$$

3. If  $y$  is the solution of the differential equation  $y^3 \frac{dy}{dx} + x^3 = 0$ ,  $y(0) = 1$ , the value of  $y(-1)$  is

(A) -2

(B) -1

(C) 0

(D) 1

[Ans. C]\*

$$y^3 \frac{dy}{dx} = -x^3$$

$$y^3 dy = -x^3 dx$$

$$\int y^3 dy = - \int x^3 dx$$

$$\frac{y^4}{4} = -\frac{x^4}{4} + C$$

$$\frac{x^4 + y^4}{4} = C$$

$$y(0) = 1,$$

$$\frac{0 + 1}{4} = C$$

$$C = \frac{1}{4}$$

$$x^4 + y^4 = 1$$

$$y^4 = 1 - x^4$$

$$y = \sqrt[4]{1 - x^4}$$

$$\text{When } x = -1$$

$$y = 0$$

4. Select the correct statement for 50% reaction stage in a steam turbine.
- (A) The rotor blade is symmetric
  - (B) The stator blade is symmetric
  - (C) The absolute inlet flow angle is equal to absolute exit flow angle.
  - (D) The absolute exit flow angle is equal to inlet angle of rotor blade.

[Ans. D]\*

For 50% reaction stage,  $\alpha = \phi$  and  $\beta = \theta$

5. For an ideal gas with constant properties undergoing a quasi-static process, which one of the following represents the change of entropy ( $\Delta s$ ) from state 1 to 2?

(A)  $\Delta s = C_p \ln \left[ \frac{T_2}{T_1} \right] - R \ln \left[ \frac{P_2}{P_1} \right]$

(B)  $\Delta s = C_v \ln \left[ \frac{T_2}{T_1} \right] - C_p \ln \left[ \frac{P_2}{P_1} \right]$

(C)  $\Delta s = C_p \ln \left[ \frac{T_2}{T_1} \right] - C_v \ln \left[ \frac{P_1}{P_2} \right]$

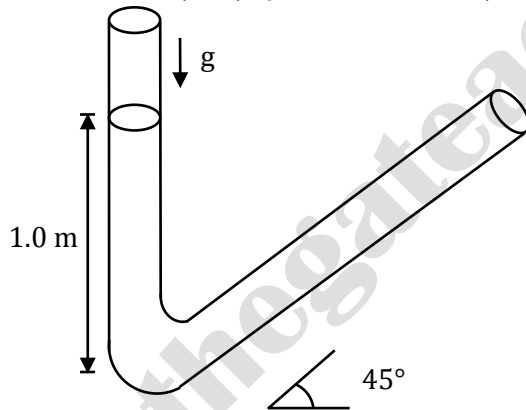
(D)  $\Delta s = C_p \ln \left[ \frac{T_2}{T_1} \right] + R \ln \left[ \frac{V_1}{V_2} \right]$

[Ans. A]

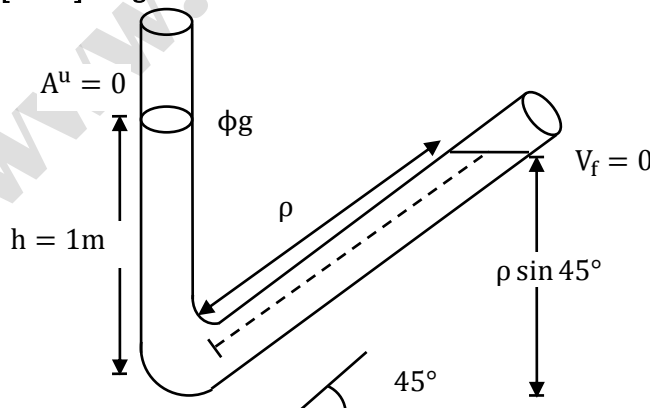
$$S_2 - S_1 = C_p \ln \left[ \frac{T_2}{T_1} \right] - R \ln \left[ \frac{P_2}{P_1} \right]$$

$$S_2 - S_1 = C_v \ln \left[ \frac{T_2}{T_1} \right] - R \ln \left[ \frac{V_2}{V_1} \right]$$

6. A ball is dropped from the rest from a height of 1 m in a frictionless tube as shown in the figure. If the tube profile is approximated by two straight lines (ignoring the curved portion), the total distance travelled (in m) by the ball is \_\_\_\_\_ (correct to two decimal places)



[Ans. \*] Range: 2.40 to 2.42



$$[PE + KE]_A = [PE + KE]_B$$

As  $KE_A = 0$  (Ball is dropped)



$KE_B =$  Ball finally comes to rest

$$mg \cdot h = mg \cdot \rho \cdot \sin 45^\circ$$

$$h = \frac{\rho}{\sqrt{2}} \Rightarrow \rho = \sqrt{2} \cdot h$$

$$\begin{aligned} \text{Total distance travel} &= h + \rho \\ &= h + \sqrt{2}h \\ &= h(\sqrt{2} + 1) \end{aligned}$$

Total distance travelled = 2.414 m

7. Feed rate in slab milling operation is equal to  
 (A) rotation per minute (rpm)  
 (B) product of rpm and number of teeth in the cutter  
 (C) product of rpm, feed per tooth and number of teeth in the cutter  
 (D) product of rpm, feed per tooth and number of teeth in contact

**[Ans. C]**

Feed rate in slab milling operation its equal to

$$f_r \left( \frac{\text{mm}}{\text{min}} \right) = f_t \left( \frac{\text{mm}}{\text{tooth}} \right) \times z \times N$$

8. The Fourier cosine series for an even function  $f(x)$  is given by

$$f(x) = a_0 + \sum_{n=1}^{\infty} a_n \cos(nx)$$

The value of the coefficient  $a_2$  for the function  $f(x) = \cos^2(x)$  in  $[0, \pi]$  is

- (A) -0.5 (B) 0.0  
 (C) 0.5 (D) 1.0

**[Ans. C]\***

$$\cos^2 x = \frac{1 + \cos 2x}{2}$$

$$f(x) = \frac{1}{2} + \frac{\cos 2x}{2}$$

$$f(x) = \frac{a_0}{2} + \sum_{n=1}^{\infty} a_n \cdot \cos nx$$

$$a_0 = 1$$

$$a_1 = 0,$$

$$a_2 = \frac{1}{2}$$

9. The minimum axial compressive load  $P$ , required to initiate buckling for a pinned-pinned slender column with bending stiffness  $EI$  and length  $L$  is

- (A)  $P = \frac{\pi^2 EI}{4L^2}$  (B)  $P = \frac{\pi^2 EI}{L^2}$   
 (C)  $P = \frac{3\pi^2 EI}{4L^2}$  (D)  $P = \frac{4\pi^2 EI}{L^2}$

**[Ans. B]\***

For both ends hinged bucking load,

$$P = \frac{\pi^2 EI}{L^2}$$

10. A local tyre distributor expects to sell approximately 9600 steel belted radial tyres next year. Annual carrying cost is Rs. 16 per tyre and ordering cost is Rs. 75. The economic order quantity of the tyres is

(A) 64 (B) 212  
(C) 300 (D) 1200

**[Ans. C]**

$$D=9600$$

$$C_h = R_0 \text{ 16/year}$$

$$C_o = R_1 \text{ 75/order}$$

$$Eo\theta = \sqrt{\frac{2DC_o}{C_h}} = 300$$

11. The arrival of customers over fixed time intervals in a bank follow a Poisson distribution with an average of 30 customer's hour. The probability that the time between successive customer arrival is between 1 and 3 m minutes is\_\_\_ (correct to two decimal places).

**[Ans. \*] Range: 0.36 to 0.40**

Given average customers per hour  $\lambda = \frac{30}{\text{hr}} = \frac{1}{2}$  Per min

Let X: is random variable which indicates arrival of customers over fixed time interval

$$P(1 \leq x \leq 3) = \underline{\hspace{2cm}}$$

$$= P(x = 1) + P(x = 2) + P(x = 3)$$

$$= \frac{\lambda^1 e^{-\lambda}}{1!} + \frac{\lambda^2 e^{-\lambda}}{2!} + \frac{\lambda^3 e^{-\lambda}}{3!}$$

$$= e^{-\lambda} \left[ \lambda + \frac{\lambda^2}{2} + \frac{\lambda^3}{6} \right] \frac{30 \times 30 \times 30}{6}$$

$$= e^{-1/2} \left[ \frac{1}{2} + \frac{\left(\frac{1}{2}\right)^2}{2} + \frac{\left(\frac{1}{2}\right)^3}{6} \right]$$

$$= 0.39$$

12. The preferred option for holding an odd shaped work piece in a centre lathe is

(A) live and dead centres (B) three jaw chuck  
(C) lathe dog (D) four jaw chuck

**[Ans. D]**

In four-jaw chuck, each jaw can move independently, so any irregular surface could be effectively centred.

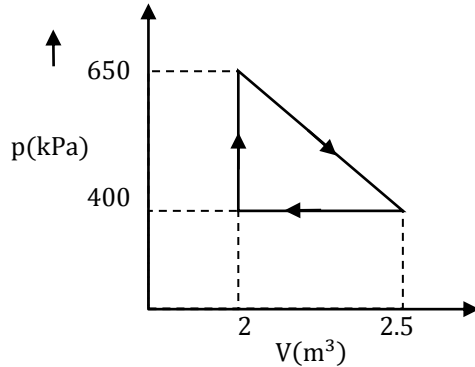
13. During solidification of a pure molten metal, the grains in the casting near the mould wall are

(A) coarse and randomly oriented  
(B) fine and randomly oriented  
(C) Fine and ordered  
(D) Coarse and ordered

**[Ans. B]**

The solidification process begins at the interface of mold and molten over the entire outer skin of the casting. This rapidly cooling action causes the grains in the skin to become fine and randomly oriented.

14. An engine operates on the reversible cycle as shown in the figure. The work output from the engine (in kJ/cycle) is \_\_\_\_ (correct to two decimal places).



[Ans. \*] Range: 62.00 to 63.00

WO/cycle = Area enclosed by the cycle

$$= \frac{1}{2} (2.5 - 2)(650 - 400)$$

$$= 62.5 \text{ kJ}$$

15. The divergence of the vector field  $\vec{u} = e^x(\cos y\hat{i} + \sin y\hat{j})$  is  
 (A) 0 (B)  $e^x \cos y + e^x \sin y$   
 (C)  $2e^x \cos y$  (D)  $2e^x \sin y$

[Ans. C]

Divergence of vector field  $\vec{u} = \nabla \cdot \vec{u}$

$$\nabla = i \frac{\partial}{\partial x} + j \frac{\partial}{\partial y} + k \frac{\partial}{\partial z}$$

$$\nabla \cdot \vec{u} = e^x \cos y = e^x \cos y$$

$$= 2e^x \cos y$$

16. Fatigue life of a material for a full reversed loading condition is estimated from  $\sigma_a = 1100 N^{-0.15}$ , where  $\sigma_a$  is the stress amplitude in MPa and N is the failure life in cycles. The maximum allowable stress amplitude (in MPa) for a life of  $1 \times 10^5$  cycles under the same loading condition is \_\_\_\_ (correct to two decimal places)

[Ans. \*] Range: 190.00 to 200.00

$$\sigma_a = 1100 N^{-0.15}$$

$$N = 1 \times 10^5$$

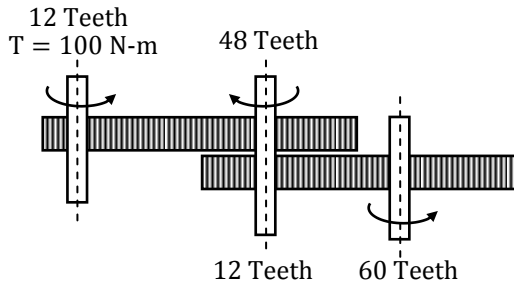
$$\Rightarrow \sigma_a = (1100)(10^5)^{-0.15}$$

$$N = 1 \times 10^5$$

$$\Rightarrow \sigma_a = (1100)(10^5)^{-0.15}$$

$$= 195.610 \text{ MPa}$$

17. A frictionless gear train is shown in the figure. The leftmost 12-teeth gear is given a torque of 100 N-m. The output torque from the 60-teeth gear on the right in N-m is



- (A) 5 (B) 20  
(C) 500 (D) 2000

[Ans. D]

Since it is frictionless gear train input power = output power

$$P_{I/P} = P_{O/P}$$

$$T_1 \omega_1 = T_4 \omega_4$$

$$\frac{T_4}{T_1} = \frac{\omega_1}{\omega_4}$$

$$(1, 2) \frac{\omega_1}{\omega_2} = \frac{T_2}{T_1} \dots \textcircled{1}$$

$$(3, 4) \frac{\omega_3}{\omega_4} = \frac{T_4}{T_3} \dots \textcircled{2}$$

From  $\textcircled{1}$  and  $\textcircled{2}$

$$\frac{\omega_1}{\omega_2} * \frac{\omega_3}{\omega_4} = \frac{T_2 T_4}{T_1 T_3}$$

$$\frac{\omega_1}{\omega_4} = \frac{(48)(60)}{(12)(12)} = 20$$

$$\frac{T_4}{T_1} = 20$$

$$T_4 = (20)(100) = 2000 \text{ Nm}$$

18. Metal removal in-electric discharge machining takes place through  
(A) Ion displacement (B) Melting and vaporization  
(C) Corrosive reaction (D) Plastic shear

[Ans. B]

In EDM, the metal removal takes place due to melting and evaporation of the work piece material.

19. The viscous laminar flow of air over a flat plate results in the formation of a boundary layer. The boundary layer thickness at the end of the plate of length L is  $\delta_L$ . When the plate length is increased to twice its original length, the percentage change in laminar boundary layer thickness at the end of the plate (with respect to  $\delta_L$ ) is \_\_\_ (correct to two decimal places).

[Ans. \*] Range: 41.30 to 41.50

For laminar boundary layer

$$\delta_x = \frac{5x}{\sqrt{R_{ex}}} \text{ by blassius solution}$$

Therefore

$$\Rightarrow \delta_x \propto \sqrt{x}$$

$$\Rightarrow \frac{\delta_1}{\delta_2} = \sqrt{\frac{x_1}{x_2}}$$

For trailing edge

$$\Rightarrow \frac{\delta_1}{\delta_2} = \sqrt{\frac{L_1}{L_2}}$$

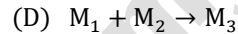
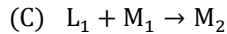
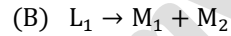
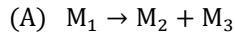
$$\text{i. e., } \delta_2 = \delta_1 \left( \sqrt{\frac{L_2}{L_1}} \right)$$

But  $L_2 = 2L_1$

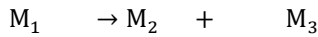
$$\Rightarrow \delta_2 = 1.414 \delta_1$$

Thus  $\delta$  increase by 41.44

20. Denoting L as liquid and M as solid in a phase-diagram with the subscripts representing different phase, a eutectoid reaction is described by



[Ans. A]\*



$\gamma$  – iron  $\alpha$  – ferrite cementite ( $Fe_3C$ )

21. Pre-tensioning of a bolted joint is used to

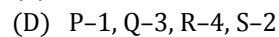
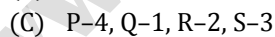
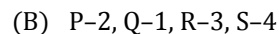
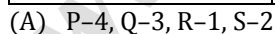
- (A) Strain harden the bolt head  
(B) Decreases stiffness of the bolted joint  
(C) Increase stiffness of the bolted joint  
(D) Prevent yielding of the thread root

[Ans. C]

Pre-tensioning is the tightening of a fastener to its maximum tension before the full load is placed on the material. Pre-tensioning of a bolted joint is used to increase stiffness of the bolted joint.

22. Match the following products with the suitable manufacturing process.

Product	Manufacturing process
P. Toothpaste tube	1. Centrifugal casting
Q. Metallic pipes	2. Blow moulding
R. Plastic bottles	3. Rolling
S. Threaded bolts	4. Impact extrusion



[Ans. C]

Tooth paste tube – Impact extrusion

Metallic pipes – centrifugal

Plastic bottle – Blow moulding

Threaded bolts – Rolling

23. Consider a function 'u' which depends on position x and time t. The partial differential equation

$$\frac{\partial u}{\partial t} = \frac{\partial^2 u}{\partial x^2}$$

is known as the

- (A) Wave equation (B) Heat equation  
(C) Laplace's equation (D) Elasticity equation

**[Ans. B]**

$\frac{\partial u}{\partial t} = \frac{\partial^2 u}{\partial x^2}$  is a representation of heat equation

24. The peak wavelength of radiation emitted by black body at a temperature of 2000 K is 1.45  $\mu\text{m}$ . If the peak wavelength of emitted radiation changes to 2.90  $\mu\text{m}$ . then the temperature (in K) of the black body is

- (A) 500 (B) 1000  
(C) 4000 (D) 8000

**[Ans. B]**

By Wich's displacement law

$$\lambda_m T = C$$

Where  $\lambda_m$  is the peak wavelength corresponding to temperature T in k

**For state 1**

$$\lambda_1 T_1 = C$$

**For state 2**

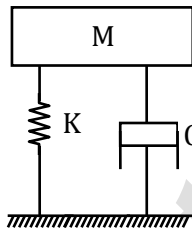
$$\lambda_2 T_2 = C$$

$$\Rightarrow \lambda_1 T_1 = \lambda_2 T_2$$

$$\Rightarrow T_2 = \lambda_1 \left(\frac{T_1}{\lambda_2}\right) \left(\frac{\lambda_1}{\lambda_2}\right) T_1$$

$$\Rightarrow T_2 = \left(\frac{1.45}{2.90}\right) \times 2000 = 1000 \text{ k}$$

25. In a single degree of freedom under-damped spring mass-damper system as shown in the figure, and additional damper is added in parallel such that the system still remains under-damped. Which one of the following statements is ALWAYS true?



- (A) Transmissibility will increase  
(B) Transmissibility will decrease.  
(C) time period of free oscillations will increase.  
(D) Time period of free oscillation will decrease.

**[Ans. C]**

$$\omega_d = \omega_n \sqrt{1 - \xi^2}$$

$$\omega_d < \omega_n$$

$$\text{(Time period of damped vibration)} T_d = \frac{2\pi}{\omega_d}$$

$$\text{(Time period of natural vibrations)} T_n = \frac{2\pi}{\omega_n}$$

$$T_d > T_n$$

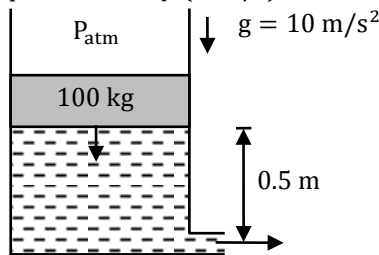
$$\omega_d = \omega_n \sqrt{1 - \xi^2}$$

$$T_d = \frac{2\pi}{\omega_d}$$

By adding additional temperature  $\xi$  increases. Implies  $\omega_d$  decreases impels time period increases

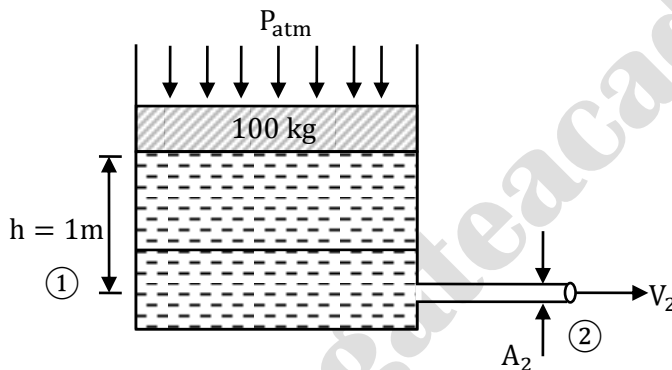
**Q.26 - Q.55 Carry Two Mark each.**

26. A frictionless circular piston of area  $10^{-2} \text{ m}^2$  and mass 100 kg sinks into a cylindrical container of the same area filled with water of density  $1000 \text{ kg/m}^3$  as shown in the figure. The container has a hole of area  $10^{-3} \text{ m}^2$  at the bottom that is open to the atmosphere. Assuming there is no leakage from the edges of the piston and considering water to be incompressible the magnitude of the piston velocity (in m/s) at the instant shown is \_\_\_\_\_ (correct to three decimal places).



[Ans. \*]Range: 1.400 to 1.500

Applying Bernoulli's equation between section ① and section ②



$$\frac{p_1}{\rho_g} + \frac{V_1^2}{2g} + y_1 = \frac{p_2}{\rho_g} + \frac{V_2^2}{2g} + y_2$$

But  $y_1 = y_2$

$$p_1 = p_{atm} + \frac{W}{A_1} + \rho_g h$$

$$p_r = p_{atm}$$

$$\Rightarrow \frac{W}{\rho_g A_1} + h + \frac{V_1^2}{2g} = \frac{V_2^2}{2g}$$

$$\Rightarrow \frac{M}{\rho A_1} + h = \frac{V_2^2 - V_1^2}{2g}$$

Using Continuity equation

$$A_1 V_1 = A_2 V_2$$

$$V_2 = \frac{A_1 V_1}{A_2}$$

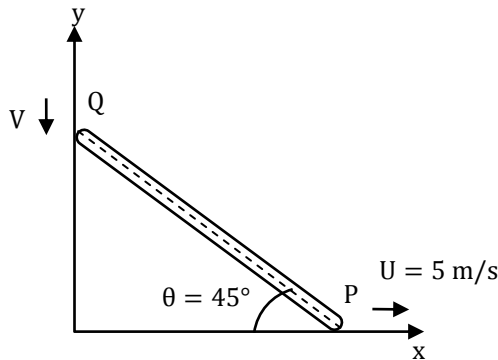
$$\Rightarrow \frac{M}{\rho A_1} + h = \left[ \frac{A_1^2}{A_2^2} - 1 \right] \frac{V_1^2}{2g}$$

$$\Rightarrow \left[ \frac{100}{1000 \times 10^{-2}} + 1 \right] = \left[ \left( \frac{10^{-2}}{10^{-3}} \right)^2 - 1 \right] \frac{V_1^2}{2g}$$

$$\Rightarrow 11 = 99 \frac{V_1^2}{2g}$$

$$\Rightarrow V_1 = \left[ \frac{11 \times 2 \times 9.81}{99} \right]^{\frac{1}{2}} \quad V_1 = 1.476 \text{ m/sec}$$

27. A rigid rod of length 1 m is resting at an angle  $\theta = 45^\circ$  as shown in the figure. The end P is dragged with a velocity of  $U = 5 \text{ m/s}$  to the right. At the instant shown, the magnitude of the velocity  $V$  (in m/s) of point Q as it moves along the wall without losing contact is

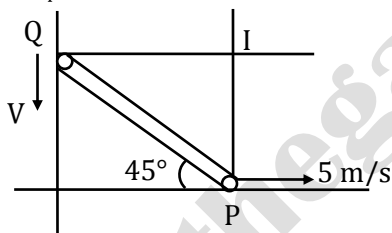


- (A) 5 (B) 6  
(C) 8 (D) 10

**[Ans. A]**

$$\left. \begin{array}{l} V_q \times (IQ) \\ V_p \times (IP) \end{array} \right\}$$

$$\Rightarrow \frac{V_q}{V_p} = \frac{IQ}{IP}$$



$$\frac{V_q}{5} = 1$$

$$V_q = 5 \text{ m/s}$$

28. A steel wire is drawn from an initial diameter ( $d_i$ ) of 10 mm to a final diameter ( $d_f$ ) of 7.5 mm. The half cone angle ( $\alpha$ ) of the die is  $5^\circ$  and the coefficient of friction ( $\mu$ ) between the die and the wire is 0.1. The average of the initial and final yield stress  $Y_{\text{avg}}$  [ $(\sigma_Y)_{\text{avg}}$ ] is 350 MPa. The equation for drawing stress  $\sigma_f$  (in MPa) is given as:

$$\sigma_f = (\sigma_Y)_{\text{avg}} \left\{ 1 + \frac{1}{\mu \cot \alpha} \right\} \left[ 1 - \left( \frac{d_f}{d_i} \right)^{2\mu \cot \alpha} \right]$$

The drawing stress (in MPa) required to carry out this operation is \_\_\_\_\_ (correct to two decimal places).

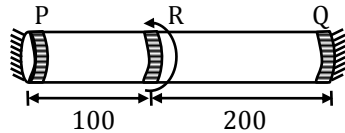
**[Ans. \*] Range: 315.00 to 317.00**

$$= 350 \times \left( 1 + \frac{1}{0.1 \times \cot 5} \right) \left( 1 - \left( \frac{7.5}{10} \right)^{2 \times 0.1 \times \cot 5} \right)$$



= 316.24 MPa

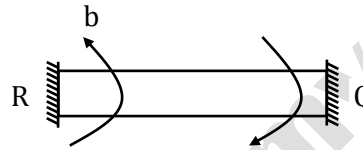
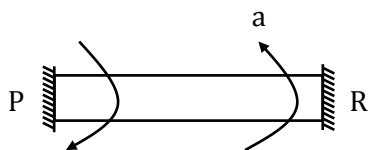
29. A bar of a circular cross-section is clamped at ends P and Q as shown in the figure. A torsional moment  $T = 150 \text{ Nm}$  is applied at distance of 100 mm from P. The torsional reaction ( $T_P, T_Q$ ) in Nm at the ends P and Q respectively are



(All dimensions are in mm)

- (A) (50, 100) (B) (75, 75)  
(C) (100, 50) (D) (120, 30)

[Ans. C]



$$T = a + b$$

$$\theta_{PR} = \theta_{RQ}$$

$$\Rightarrow \left(\frac{aL}{GJ}\right)_{PR} = \left(\frac{bL}{GJ}\right)_{RQ} \Rightarrow a(0.1) = b(0.2)$$

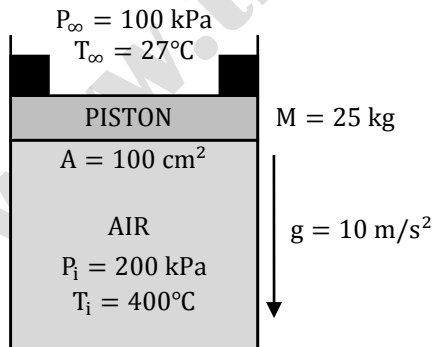
$$a = 2b$$

$$J = 3b$$

$$b = \frac{J}{3} = 50$$

$$a = \frac{2J}{3} = 100$$

30. Air is held inside a non-insulated cylinder using a piston (mass  $M = 25 \text{ kg}$  and area  $A = 100 \text{ cm}^2$ ) and stoppers (of negligible area), as shown in the figure. The initial pressure  $P_i$  and temperature  $T_i$  of air inside the cylinder are 200 kPa and  $400^\circ\text{C}$  respectively. The ambient pressure  $P_\infty$  and temperature  $T_\infty$  are 100 kPa and  $27^\circ\text{C}$ , respectively. The temperature of the air inside the cylinder ( $^\circ\text{C}$ ) at which the piston will begin to move is \_\_\_\_\_ (correct to two decimal places).



[Ans. \*] Range : 145.00 to 150.00

Since the cylinder is not insulated it loses heat to the ambient air. As it loses heat its temperature decrease and due to this pressure decrease  $\Rightarrow$  piston starts moving down, when the pressure of air is strictly less than outside pressure.

$$P_2 = \left[ P_{\text{atm}} + \frac{mg}{A} \right]$$

$$\frac{P_1 V_1}{T_1} = \frac{P_2 V_2}{T_2}$$

At the instant when piston started moving  $V_1 = V_2$

$$\frac{P_1}{T_1} = \frac{P_2}{T_2}$$

$$T_2 = \left[ \frac{P_2}{P_1} \right] T_1$$

$$T_2 = \left[ \frac{125}{200} \right] \times (273 + 400)$$

$$T_2 = 420.625 \text{ K}$$

$$T_2 = 147.625^\circ\text{C}$$

31. A test is conducted on a one-fifth scale model of a Francis turbine under a head of 2 m and volumetric flow rate of  $1 \text{ m}^3/\text{s}$  at 450 rpm. Take the water density and the acceleration due to gravity as  $103 \text{ kg/m}^3$  and  $10 \text{ m/s}^2$ , respectively. Assume no losses both in model and prototype turbines. The power (in MW) of a full sized turbine while working under a head of 30 m is \_\_\_\_\_ (correct to two decimal places).

**[Ans. \*] Range: 28.85 to 29.25**

For model

$$\rho_m = \rho_g QH = 1000 \times 10 \times 1 \times 2 = 20 \text{ kW}$$

$$\frac{H}{D^2 N^2} \Big|_p = \frac{H}{D^2 N^2} \Big|_M$$

$$\Rightarrow N_p = \sqrt{\frac{H_p}{H_M}} \times \frac{D_M}{D_p} \times N_M = \sqrt{\frac{30}{2}} \times \frac{1}{5} \times 450$$

$$= 348.56 \text{ Npm}$$

$$\frac{P}{D^5 N^3} \Big|_p = \frac{P}{D^5 N^3} \Big|_M$$

$$\Rightarrow P_p = P_M \times \frac{D_p^5}{D_M^5} \times \frac{N_p^3}{N_M^3}$$

$$= 20 \times 5^5 \times \frac{348.56^3}{450^3} = 29.05 \text{ Mw}$$

32. Following data correspond to an orthogonal turning of a 100 mm diameter rod on a lathe. Rake angle:  $+15^\circ$ ; Uncut chip thickness: 0.5 mm; nominal chip thickness after the cut: 1.25 mm. The shear angle (in degrees) for this process is \_\_\_\_\_ (correct to two decimal places).

**[Ans. \*] Range: 22.00 to 24.00**

$$t_1 = 0.5$$

$$t_2 = 1.25$$

$$r = \frac{t_1}{t_2} = 0.4$$

$$\tan \phi = \frac{r \cos \alpha}{1 - r \sin \alpha}$$

$$\phi = 23.315^\circ$$

33. For sand-casting a steel with dimensions  $80 \text{ mm} \times 120 \text{ mm} \times 20 \text{ mm}$  a cylindrical riser has to be designed. The height of the riser is equal to its diameter. The total solidification time for the casting is 2 minutes. In Chvorinov's law for the estimation of the total solidification time, exponent is to be

taken as 2. For a solidification time of 3 minutes in the riser, the diameter (in mm) of the riser is \_\_\_\_ (correct to two decimal places).

[Ans. \*] Range: 51.50 to 52.00

$$\tau_c = C_m \left(\frac{V}{A}\right)^2$$

$$2 = C_m \times \left(\frac{80 \times 120 \times 20}{2 \times (9600 + 2400 + 1600)}\right)^2$$

$$C_m = \frac{2}{49.82} = 0.04$$

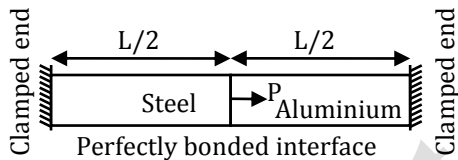
$$\tau_r = C_m \left(\frac{V}{A}\right)^2$$

$$3 = 0.04 \times \left[\frac{\frac{\pi}{4} D^3}{\frac{\pi}{2} D^2 + \pi D^2}\right]$$

$$\left(\frac{3}{0.04}\right)^{1/2} = \frac{D}{6} = 8.66$$

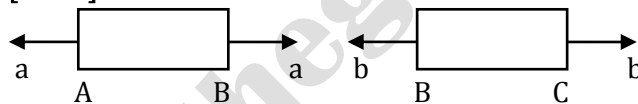
$$D = 51.96 \text{ mm}$$

34. A bimetallic cylindrical bar of cross-sectional area  $1 \text{ m}^2$  is made by bonding steel (young's modulus = 210 GPa) and Aluminum (Young's modulus = 70 GPa) as shown in the figure. To maintain tensile axial strain of magnitude  $10^{-6}$  in steel bar and compressive axial strain of magnitude  $10^{-6}$  in Aluminum bar, the magnitude of the required force P (in kN) along the indicated direction is



- (A) 70 (B) 140  
(C) 210 (D) 280

[Ans. D]



$$a - b = P \quad \dots \textcircled{1}$$

$$\left(\frac{aL}{AE}\right) + \left(\frac{bL}{AE}\right) = 0$$

$$\frac{a\left(\frac{L}{2}\right)}{(1)(210)} + \frac{(b)\left(\frac{L}{2}\right)}{(1)(70)} = 0$$

$$\Rightarrow \left(\frac{a}{21}\right) + \left(\frac{b}{7}\right) = 0$$

$$\Rightarrow \frac{a}{3} + b = 0 \quad \dots \dots \textcircled{2}$$

From  $\textcircled{1}$  and  $\textcircled{2}$

$$a + \frac{a}{3} = P \Rightarrow \left(\frac{4a}{3}\right) = P$$

$$a = \left(\frac{3P}{4}\right)$$

$$b = \left(\frac{3P}{4}\right) - P = -\frac{P}{4}$$

$$\text{Also, } \frac{aL}{AE} = (10^{-6} \times L)$$

$$\left(\frac{3P}{4}\right) \left(\frac{L}{AE}\right) = (10^{-6} \times L)$$

$$\left(\frac{3}{4}\right) \frac{(P)}{AE} = 10^{-6} \Rightarrow P = \frac{4 \times 10^{-6} \times (1) \times 210 \times 10^9}{3}$$

$$= 280 \times 10^3 \text{ N}; P = 280 \text{ kN}$$

35. A vehicle powered by a spark ignition engine follows air standard Otto cycle ( $\gamma = 1.4$ ). The engine generates 70 kW while consuming 10.3 kg/hr of fuel. The calorific value of fuel is 44,000 kJ/kg. The compression ratio is \_\_\_\_\_ (correction to two decimal places).

[Ans. \*] Range: 7.40 to 7.80

$$\text{For otto cycle } \eta = 1 - \frac{1}{r_c^{\gamma-1}} = \frac{W_D}{\Phi_s}$$

$$W_D = 70 \text{ kW}$$

$$Q_s = (\dot{m}_f)(C.V) = (10.3)(44000)$$

$$Q_s = 453200 \text{ kJ/hr}$$

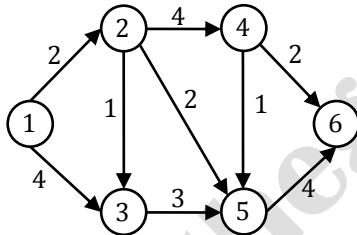
$$Q_s = 125.89 \text{ kWatt}$$

$$\eta = \frac{W_D}{Q_s} = \frac{70}{125.89} = 0.556$$

$$\eta = 1 - \frac{1}{r_c^{\gamma-1}} = 0.556$$

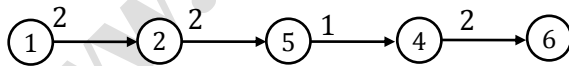
$$r_c = 7.614$$

36. The arc lengths of a directed graph of a project are as shown in the figure. The shortest path length from node 1 to node 6 is \_\_\_\_\_



[Ans. \*] Range: 7 to 7

Shortest path is



Shortest length=7

37. Given the ordinary differential equation

$$\frac{d^2y}{dx^2} + \frac{dy}{dx} - 6y = 0$$

With  $y(0) = 0$  and  $\frac{dy}{dx}(0) = 1$ , the value of  $y(1)$  is \_\_\_\_\_ (correct to two decimal places).

**[Ans. \*] Range: 1.45 to 1.48\***

$$(D^2 + D - 6)y = 0$$

$$y(0) = 0,$$

$$y(0) = 1$$

$$(D + 3)(D - 2)y = 0$$

$$D = 2, -3$$

$$C.F = C_1 e^{2x} + C_2 e^{-3x}$$

$$y = C_1 e^{2x} + C_2 e^{-3x}$$

$$y(0) = 0$$

$$\text{So, } 0 = C_1 + C_2 \dots (i)$$

$$\frac{dy}{dx} = 2C_1 e^{2x} - 3C_2 e^{-3x}$$

$$y(0) = 1,$$

$$1 = 2C_1 - 3C_2 \dots (ii)$$

From equation (i) and (ii)

$$C_1 = \frac{1}{5},$$

$$C_2 = -\frac{1}{5}$$

$$y = \frac{1}{5} e^{2x} - \frac{1}{5} e^{-3x}$$

When,  $x = 1$

$$y(1) = \frac{e^2 - e^{-3}}{5} = 1.4678$$

38. In a cam-follower, follower rises by  $h$  as the cam rotates by  $\delta$  (radians) at constant angular velocity  $\omega$  (radians/s). The follower is uniformly accelerating during the first half of the rise period and it is uniformly decelerating in the latter half of the rise period. Assuming that the magnitude of the acceleration and deceleration are same, the maximum velocity of the follower is

(A)  $\frac{4h\omega}{\delta}$

(B)  $h\omega$

(C)  $\frac{2h\omega}{\delta}$

(D)  $2 h\omega$

**[Ans. C]**

The follower motion is parabolic motion.

$$y = C\theta^2$$

For first half of rise follower is having uniform acceleration.

Therefore maximum velocity of follower will occur at the mid-point of rise.

$$\text{When } \theta = \frac{\theta_{ri}}{2}; y = \frac{h}{2}$$

$$\left(\frac{h}{2}\right) = C \left(\frac{\theta_{ri}}{2}\right)^2$$

$$C = \frac{2h}{\theta_{ri}^2}$$

$$y = 2h \left( \frac{\theta}{\theta_{ri}} \right)^2$$

$$\frac{dy}{dt} = \frac{2h}{\theta^2 r_i} \left[ 2\theta \frac{d\theta}{dt} \right]$$

$$V = \frac{4h\omega\theta}{\theta^2 r_i}$$

$$V_{\max} \text{ occurs at } \theta = \frac{\theta_{ri}}{2}$$

$$V_{\max} = \frac{2h\omega}{\theta_{ri}} = \frac{2h\omega}{\delta}$$

39. Let  $X_1$  and  $X_2$  be two independent exponentially distributed random variables with means 0.5 and 0.25, respectively. Then  $Y = \min(X_1, X_2)$  is  
 (A) exponentially distributed with mean 1/6  
 (B) exponentially distributed with mean 2  
 (C) normally distributed with mean 3/4  
 (D) normally distributed with mean 1/6

**[Ans. A]\***

$$\text{Mean}(x_1) = 0.5$$

$$\frac{1}{\lambda_1} = 0.5$$

$$\lambda_1 = \frac{1}{0.5} = 2$$

$$\text{Mean}(x_2) = 0.25$$

$$\frac{1}{\lambda_2} = 0.25$$

$$\lambda_2 = \frac{1}{0.25} = 4$$

$$y = \text{mean}(x_1, x_2)$$

$$\text{mean}(y) = \frac{1}{\lambda_1 + \lambda_2} = \frac{1}{2 + 4} = \frac{1}{6}$$

40. A bar is subjected to a combination of a steady load of 60 kN and a load fluctuating between  $-10$  kN and 90 kN. The corrected endurance limit of the bar is 150 MPa, the yield strength of the material is 480 and the ultimate strength of the material is 600 MPa. The bar cross-section is square with side  $a$ . If the factor of safety is 2, the value of  $a$  (in mm), according to the modified Goodman's criterion is \_\_\_\_\_ (correct to two decimal places).

**[Ans. \*] Range: 31.60 to 31.65**

$$P_{\max} = 60 + 90 = 150 \text{ kN}$$

$$P_{\min} = 60 - 10 = 50 \text{ kN}$$

$$P_{\text{mean}} = 100 \text{ kN}$$

$$P_a = 50 \text{ kN}$$

Let, bar be of side  $a$  modified good man diagram is super imposition of both good man diagram and yield line for tensile stresses

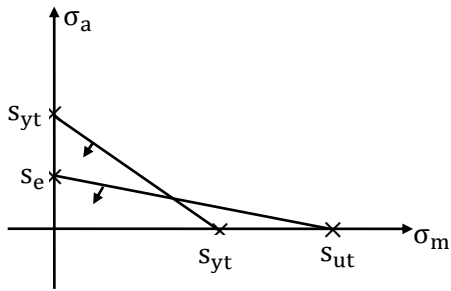
(i) Good man diagram

$$\frac{\sigma_m}{s_{ut}} + \frac{\sigma_a}{se} \leq \frac{1}{n}$$

$$10^3 \left[ \frac{(100)}{a^2 \times 600} + \frac{(50)}{(a^2) \times 150} \right] \leq \frac{1}{2}$$

$$a^2 \geq 1000$$

$\Rightarrow a \geq 31.62 \text{ mm}$



(ii) Yield line

$$\frac{\sigma_m}{S_{yt}} + \frac{\sigma_a}{S_{yt}} \leq \frac{1}{n}$$

$$10^3 \left[ \frac{100}{480 \times a^2} + \frac{50}{480} \right] \leq 1/2$$

$$a^2 \geq 625$$

$$a \geq 25 \text{ mm}$$

Hence  $a=31.62\text{mm}$

41. The problem of maximizing  $z = x_1 - x_2$  subjected to constraints  $x_1 + x_2 \leq 10, x_1 > 0, x_2 \geq 0$  and  $x_2 \leq 5$  has

(A) no solution

(B) one solution

(C) two solutions

(D) more than two solutions

**[Ans. B]**

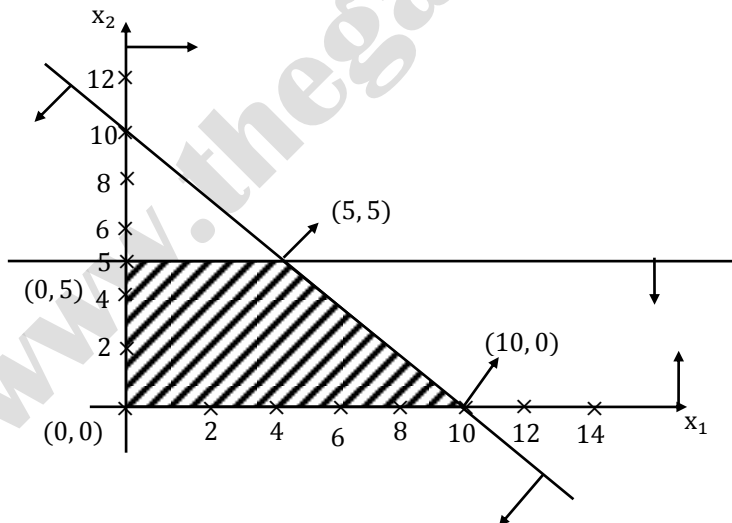
maximize  $z = x_1 - x_2$

subjected to  $x_1 + x_2 \leq 10$

$x_1 > 0$

$x_2 \geq 0$

$x_2 \leq 5$



$$z|_{(0,0)} = 0$$

$$z|_{0,5} = -5$$

$$z|_{10,0} = 10$$

$$z|_{5,5} = 0$$

$$z = 10$$

42. Let  $z$  be a complex variable. For a counterclockwise integration around a unit circle  $C$  centred at origin.

$$\oint_C \frac{1}{5z - 4} dz = A\pi i,$$

The value of  $A$  is

- (A)  $2/5$  (B)  $1/2$   
(C)  $2$  (D)  $4/5$

**[Ans. A]**

$$5z - 4 = 0$$

$z = \frac{4}{5}$  lies inside circle,

$$|z| = 1$$

$$\int \frac{1}{(5z - 4)} dz = A\pi i$$

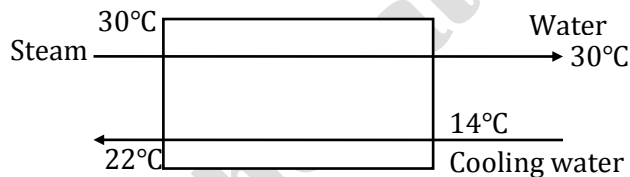
$$\frac{1}{5} \int \frac{1}{\left[z - \frac{4}{5}\right]} dz = A\pi i$$

$$\int \frac{\left(\frac{1}{5}\right)}{\left(z - \frac{4}{5}\right)} dz = 2\pi i \cdot f\left(\frac{4}{5}\right) = 2\pi i \times \left(\frac{1}{5}\right) = \frac{2}{5} \pi i$$

$$A = \frac{2}{5} = 0.4$$

43. Steam in the condenser of a thermal power plant is to be condensed at a temperature of  $30^\circ\text{C}$  with cooling water which enters the tubes of the condenser at  $14^\circ\text{C}$  and exists at  $22^\circ\text{C}$ . The total surface area of the tubes is  $50 \text{ m}^2$ , and the overall heat transfer coefficient is  $2000 \text{ W/m}^2\text{K}$ . The heat transfer (in MW) to the condenser is \_\_\_\_\_ (correct to two decimal places).

**[Ans. \*] Range: 1.14 to 1.16**



Using LMTD

$$\theta_1 = T_{Hi} - T_{Co} = 8^\circ\text{C}$$

$$\theta_2 = T_{Ho} - T_{Ci} = 16^\circ\text{C}$$

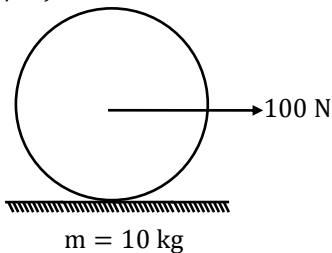
$$\Rightarrow \theta_m = \frac{\theta_1 - \theta_2}{\ln\left(\frac{\theta_1}{\theta_2}\right)} = 11.541^\circ\text{C}$$

$$\Rightarrow \dot{Q} = UA \theta_m = 1154.16 \text{ kJ/sec}$$

$$= 1.15 \text{ mw}$$



44. A force of 100 N is applied to the centre of a circular disc, of mass 10 kg and radius 1 m, resting on a floor as shown in the figure. If the disc rolls without slipping on the floor, the linear acceleration (in m/s<sup>2</sup>) of the centre of the disc is \_\_\_\_\_ (corrected to two decimal places).

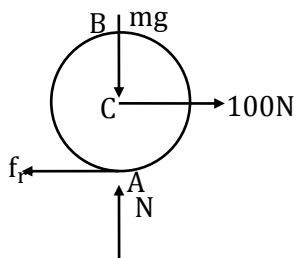


**[Ans. \*] Range: 6.60 to 6.70**

Free body diagram of disc from Newton's second law for translation

$$\sum F_x = m \times a_x$$

$$100 - f_r = m \times a \dots \textcircled{1}$$



From Newton's second law for rotation

$$\sum M_c = I_c \times \alpha$$

$$f_r \times r = \frac{mr^2}{2} \times \alpha \dots \textcircled{2}$$

$$\text{If it is pure rolling then } a = r\alpha \dots \textcircled{3} \quad \alpha = \frac{a}{r}$$

$$f_r \times r = \frac{mr^2}{2} \times \frac{a}{r} \dots \textcircled{4}$$

Putting the value of  $f_r$  in equation  $\textcircled{1}$

$$100 - \frac{ma}{2} = ma$$

$$100 = \frac{3}{2}ma$$

$$a = \frac{2}{3m} \times 100 = \frac{20}{3}$$

$$a = 6.667 \text{ m/s}^2$$

45. A welding operation is being performed with voltage = 30 V and current = 100 A. The cross-sectional area of the weld bead is 20 mm<sup>2</sup>. The work-piece and filler are of titanium for which the specific energy of melting is 14 J/mm<sup>3</sup>. Assuming a thermal efficiency of the welding process 70% the welding speed (in mm) is \_\_\_\_\_ (correct to two decimal places).

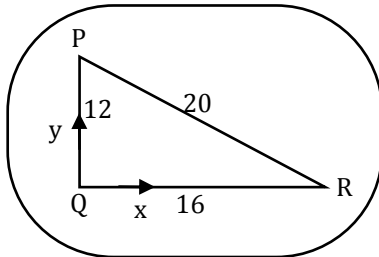
**[Ans. \*] Range : 7.49 to 7.51**

$$\text{Heat Input or SP energy} = \frac{VI \times \eta_{1+T}}{A_w \times v} \rightarrow \text{Effective power}$$

$$= \frac{30 \times 100}{20 \times v} \times 0.7 = 14$$

$$v = 7.5 \text{ mm/s}$$

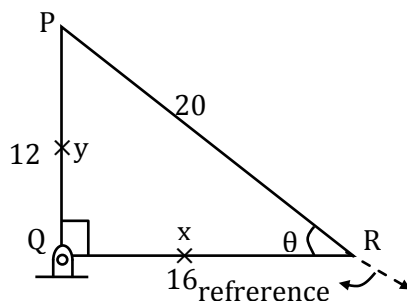
46. In a rigid body in plane motion, the point R is accelerating with respect to point P at  $10 \angle 180^\circ \text{m/s}^2$ . If the instantaneous acceleration of point Q is zero, the acceleration (in  $\text{m/s}^2$ ) of point R is



- (A)  $8 \angle 233^\circ$   
(C)  $10 \angle 217^\circ$

- (B)  $10 \angle 225^\circ$   
(D)  $8 \angle 217^\circ$

[Ans. D]



As acceleration of point Q is zero, therefore we can assume this rigid body PQR is hinged at Q at this instant

$$\vec{a}_{RP} = \vec{a}_R - \vec{a}_P$$

Is given as  $10 \text{ m/s}^2$  at an angle of  $180^\circ$ , that means only radial acceleration is there at that instant and reference is PR  $a_{RP} = (RP)\omega^2 = 10$

$$\Rightarrow 20\omega^2 = 10$$

$$\Rightarrow \omega = \frac{1}{\sqrt{2}}$$

as  $\alpha$  of whole body remains same so point R has only radial acceleration at that instant

$$a_R = QR (\omega^2) = 16 \times \frac{1}{2} = 8 \text{ m/s}^2$$

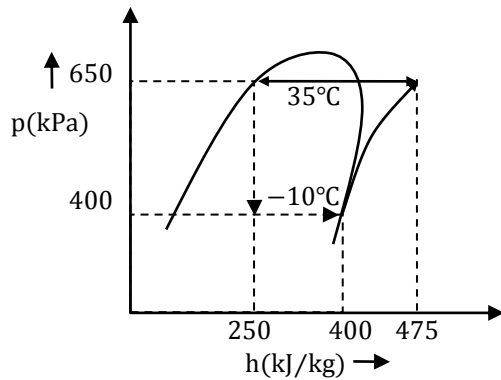
and will be in the horizontal backward direction, but our reference is along PR, so the angle of it from reference is  $(180 + \theta)$

$$\text{from } \Delta PQR \tan \theta = \frac{12}{16}$$

$$\Rightarrow \theta = 36.8698^\circ$$

$$\text{So, } 180 + 36.8698 = 216.8698 \approx 217^\circ$$

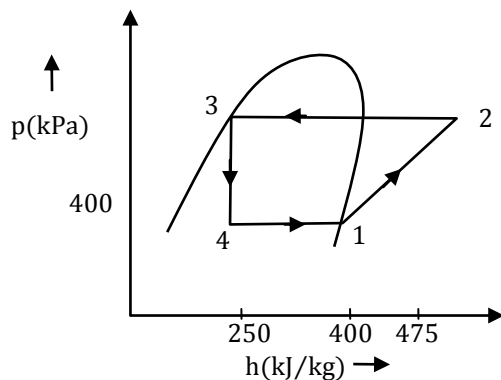
47. A standard vapor compression refrigeration cycle operating with a condensing temperature of  $35^\circ\text{C}$  and an evaporating temperature of  $-10^\circ\text{C}$  develops 15 kW of cooling. The p-h diagram shows the enthalpies at various states. If the isentropic efficiency of the compressor is 0.75, the magnitude of compressor power (in kW) is \_\_\_\_\_ (correct to two decimal places).



[Ans. \*] Range: 9.50 to 10.50

$$RE = 15 \text{ kW}$$

$$\dot{m}[h_1 - h_4] = 15$$



$$\dot{m}[400 - 250] = 15$$

$$\dot{m} = 0.1 \text{ kg}$$

$$\text{Ideal compressor work } (W_c) = h_2 - h_1$$

$$W_c = 475 - 400$$

$$W_c = 75 \text{ kJ/kg}$$

$$\text{Isentropic efficiency of compressor } (\eta_c) = \frac{(W_c)_{\text{ideal}}}{(W_c)_{\text{actual}}}$$

$$(W_c)_{\text{actual}} = \frac{75}{0.75} = 100 \text{ kJ/kg}$$

$$\text{Power of compressor} = \dot{m}(100) = 10 \text{ kW}$$

48. Ambient air is at pressure of 100 kPa. Dry bulb temperature of 30°C and 60% relative humidity. The saturation pressure of water at 30°C is 4.24 kPa. The specific humidity of air (in g/kg of dry air) is \_\_\_ (correct to two decimal places).

[Ans. \*] Range: 16.00 to 16.50

$$\text{Relative humidity } (\phi) = 0.6$$

$$\frac{P_v}{P_{vs}} = 0.6$$

$$P_v = (0.6)(4.24)$$

$$P_v = 2.544 \text{ kPa}$$

$$\begin{aligned} \text{Specific humidity } (\omega) &= \frac{0.622 P_v}{P - P_v} \\ &= \frac{0.622(2.544)}{100 - 2.544} = 0.01623 \text{ kg/kg of dry air} \end{aligned}$$

$$\omega = 16.2367 \text{ g/kg of dry air}$$

49. Taylor's tool life equation is used to estimate the life of a batch of identical HSS twist drills by drilling through holes at constant feed in 20 mm thick mild steel plates. In test 1, a drill lasted 300 holes at 150 rpm while in test 2, another drill lasted 200 holes at 300 rpm. The maximum number of holes that can be made by another drill from the above batch at 200 rpm is \_\_\_\_\_ (correct to two decimal places).

[Ans. \*] Range: 252.00 to 254.00

$$\text{Time to drill a hole} = \frac{L}{fN}$$

$$T_1 = \left( \frac{300L}{f \times 150} \right), T_2 = \left( \frac{200L}{f \times 300} \right)$$

$$T_3 = \left( y \times \frac{L}{f \times 200} \right)$$

$$V \propto N$$

$$\therefore N_1 T_1^n = N_2 T_2^n = N_3 T_3^n$$

$$\frac{N_1}{N_2} = \left( \frac{T_2}{T_1} \right)^n \Rightarrow \left( \frac{150}{300} \right) = \left( \frac{200L}{f \times 300} \times \frac{f \times 150}{300L} \right)^n$$

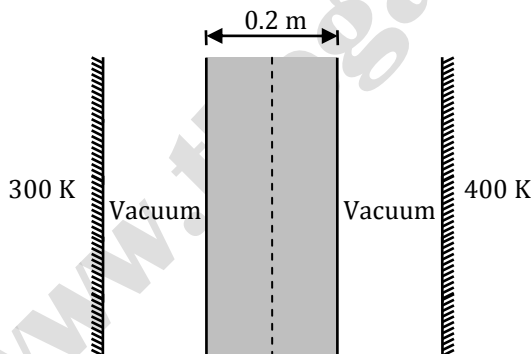
$$n = 0.63$$

$$\text{Now, } N_1 T_1^n = N_3 T_3^n$$

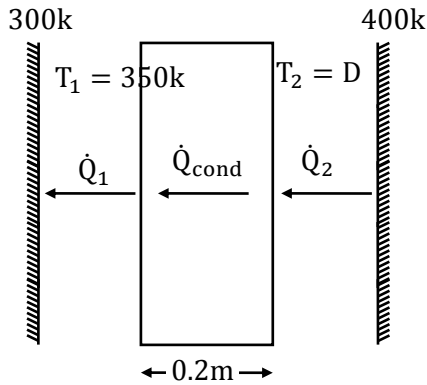
$$\left( \frac{150}{200} \right)^{1/0.63} = \frac{yL}{f \times 200} \times \frac{f \times 150}{300L}$$

$$\therefore y = 253.36$$

50. A 0.2 m thick infinite black plate having a thermal conductivity of 3.96 W/m-K is exposed to two infinite black surfaces at 300 K and 400 K as shown in the figure. At steady state, the surface temperature of the plate facing the cold side is 350 K. The value of Stefan-Boltzmann constant  $\sigma$ , is  $5.67 \times 10^{-8} \text{ W/m}^2 \text{ K}^4$ . Assuming 1-D heat conduction, the magnitude of heat flux through the plate (in  $\text{W/m}^2$ ) is \_\_\_\_\_ (correct to two decimal places)



[Ans. \*] Range: 385.00 to 395.00



As the system is at steady state

$$\Rightarrow \dot{Q}_{cond} = \dot{Q}_1$$

$$\Rightarrow \dot{Q}_{cond} = \frac{E_b(350) - E_b(300)}{R}$$

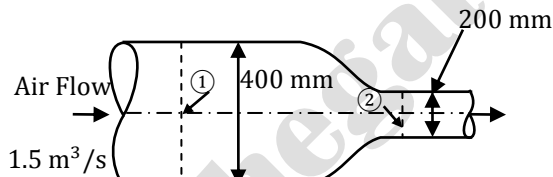
As both plates are infinitely large and black

$$R_s = \frac{1}{A}$$

$$\Rightarrow \dot{Q}_{cond} = \frac{\sigma(350)^4 - \sigma(300)^4}{\frac{1}{A}}$$

$$\Rightarrow \frac{\dot{Q}_{cond}}{A} = 391 \text{ W/m}^2$$

51. Air flows at the rate of  $1.5 \text{ m}^3/\text{s}$  through a horizontal pipe with a gradually reducing cross-section as shown in the figure. The two cross-sections of the pipe have diameters of 400 mm and 200 mm. Take the air density as  $1.2 \text{ kg/m}^3$  and assume inviscid incompressible flow. The change in pressure ( $p_2 - p_1$ ) (in kPa) between sections 1 and 2 is



(A) -1.28

(B) 2.56

(C) -2.13

(D) 1.28

[Ans. A]

As the flow is inviscid incompressible Bernoulli's equation can be applied between section ① and ②

$$\Rightarrow \frac{p_1}{\rho_g} + \frac{v_1^2}{2g} + z_1 = \frac{p_2}{\rho_g} + \frac{v_2^2}{2g} + z_2$$

Using continuity equation

$$A_1 V_1 = A_2 V_2$$

$$\Rightarrow \frac{p_2 - p_1}{\rho_g} = \frac{V_1^2 - V_2^2}{2g}$$

$$\Rightarrow (p_2 - p_1) = \frac{\rho}{2} (V_1^2 - V_2^2)$$

$$V_1 = \frac{Q}{A_1} = \frac{1.5}{\frac{\pi}{4} \times 0.4^2} = 11.93 \text{ m/sec}$$

$$V_2 = \frac{Q}{A_2} = 47.746 \text{ m/sec}$$

$$\Rightarrow (p_2 - p_1) = \frac{1.2}{2} (11.93^2 - 47.746^2)$$

$$\Rightarrow p_2 - p_1 = -1282 \text{ pa}$$

$$= -1.282 \text{ kpa}$$

52. A circular hole of 25 mm diameter and depth of 20 mm is machined by EDM process. The material removal rate (in mm<sup>3</sup>/min) is expressed as  $4 \times 10^4 IT^{-1.23}$  where  $I = 300 \text{ A}$  and the melting point of the material.  $T = 1600^\circ\text{C}$ . The time (in minutes) for machining this hole is \_\_\_\_\_ (correct to two decimal places)

[Ans. \*] Range: 7.00 to 7.30

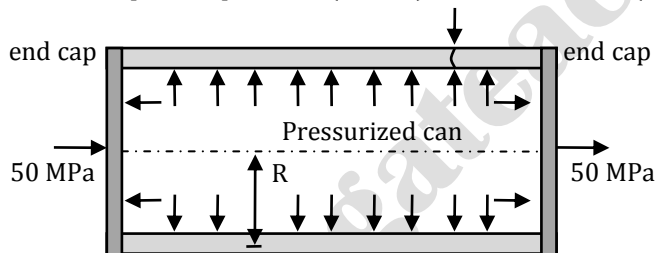
$$\text{MRR} = 4 \times 10^4 \times 300 \times 1600^{-1.23}$$

$$= 1374.40 \text{ mm}^3/\text{min}$$

$$\text{volme required to remove} = \frac{\pi}{4} \times 25^2 \times 20 = 9817.47 \text{ mm}^3$$

$$\text{machining time} = \frac{9817.47}{1374.40} = 7.14 \text{ min}$$

53. A thin-walled cylindrical can with rigid end caps has a mean radius  $R = 100 \text{ mm}$  and a wall thickness of  $t = 5 \text{ mm}$ . The can is pressurized and an additional tensile stress of 50 MPa is imposed along the axial direction as shown in the figure. Assume that the state of stress in the wall is uniform along its length. If the magnitudes of axial and circumferential components of stress in the can area equal the pressure (in MPa) inside the can \_\_\_\_\_ (correct to two decimal places)

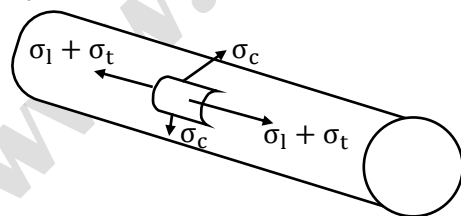


[Ans. \*] Range: 4.80 to 5.20

$$R = 100 \text{ mm}$$

$$t = 5 \text{ mm}$$

$$\sigma_t = 50 \text{ MPa}$$



$$\sigma_1 + \sigma_t = \sigma_c$$

$$\frac{PD}{4t} + \sigma_t = \frac{PD}{2t}$$

$$\frac{PR}{2t} + \sigma_t = \frac{PR}{t}$$

$$\frac{PR}{2t} + \sigma_t = \frac{PR}{t}$$

$$\frac{PR}{2t} + \sigma_t = \frac{PR}{t}$$

$$\sigma_t = \frac{PR}{2t} \Rightarrow P \left[ \frac{(\sigma_t)2t}{R} \right]$$

$$P = \left( \frac{50 \times 2 \times 5}{100} \right) = 5$$

54. For a position vector  $\vec{r} = x\hat{i} + y\hat{j} + z\hat{k}$  the norm of the vector can be defined as  $|\vec{r}| = \sqrt{x^2 + y^2 + z^2}$ .

Given a function  $\phi = \ln |\vec{r}|$ , its gradient  $\nabla\phi$  is

(A)  $\vec{r}$

(B)  $\frac{\vec{r}}{|\vec{r}|}$

(C)  $\frac{\vec{r}}{\vec{r} \cdot \vec{r}}$

(D)  $\frac{\vec{r}}{|\vec{r}|^3}$

[Ans. C]

$\phi = f(r)$

$\phi = f(r) = \ln(|\vec{r}|) = \ln(r)$

[ $\because |\vec{r}| = \sqrt{x^2 + y^2 + z^2} = r$ ]

$\nabla\phi = f'(r) \times \frac{\vec{r}}{r}$

$= \frac{1}{r} \times \frac{\vec{r}}{r} = \frac{\vec{r}}{r^2} = \frac{\vec{r}}{r \cdot r}$

55. The true stress (in MPa) versus true strain relationship for a metal is given by

$$\sigma = 1020 \varepsilon^{0.4}$$

The cross-sectional area at the start of a test (when the stress and strain values are equal to zero) is 100 mm<sup>2</sup>. The cross-sectional area at the time of necking (in mm<sup>2</sup>) is \_\_\_\_\_ (correct to two decimal places)

[Ans. \*]Range: 65.00 to 70.00\*

Given,  $\sigma = 1020 \varepsilon^{0.4}$

At UTS,  $\varepsilon_T = n = 0.4$

At UTS, neck formation starts,

$$\varepsilon_T = 0.4 = \ln \left[ \frac{A_0}{A_f} \right]$$

$$0.4 = \ln \left[ \frac{100}{A_f} \right]$$

$$\frac{100}{A_f} = e^{0.4}$$

$$A_f = \frac{100}{e^{0.4}}$$

Cross-section area at the time of necking

$$A_f = 67.032 \text{ mm}^2$$